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Application of GARCH and Value-at-Risk (VaR) Models in Stochastic Analysis of LQ45 Index Volatility

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Abstract: Stock market volatility is a crucial factor in investment decision-making. This study analyzes the volatility of the LQ45 Index, one of Indonesia's major stock indices, using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and assesses risk through the Value-at-Risk (VaR) method. The data consists of daily closing prices of the LQ45 index from 2020 to 2024. A GARCH(1,1) model is used to estimate the conditional variance dynamically, and VaR is calculated at the 95% confidence level. The results show that the GARCH(1,1) model effectively captures volatility dynamics, with the highest daily VaR recorded at 3.21% during the first quarter of 2020. The novelty of this study lies in the explicit integration of the mathematical formulation of GARCH with VaR estimation in the context of the Indonesian stock market, particularly the LQ45 index, which is rarely addressed in pure mathematical finance literature. This approach contributes to the development of stochastic financial models and provides a quantitative framework for investment risk management.

Keywords: Stock volatility; LQ45 Index; GARCH(1,1); Value-at-Risk; Risk Management

Introduction

The capital market is a vital component in the modern financial system that functions as a means of funding as well as investment. One of the instruments that is widely used as a reference by investors in Indonesia is the LQ45 Index, which is a stock index consisting of 45 stocks with high liquidity and large market capitalization on the Indonesia Stock Exchange (IDX). However, fluctuating stock price movements are a challenge for investors in managing investment risk. Therefore, a quantitative analysis model is needed that is able to measure volatility and potential risks more precisely. Stock indices such as the LQ45 not only reflect market conditions but can also be analyzed mathematically to understand the complex nature of market volatility. This analysis is crucial, as volatility represents uncertainty that can be quantified using stochastic approaches, which are highly relevant in the context of financial mathematics.

In mathematics, especially in the branches of stochastic analysis and random process theory, the study of financial variables such as stock returns opens up space for the application of various mathematical models, both deterministic and non-deterministic. The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is one of the conditional stochastic models that can be interpreted as a nonlinear dynamic system with latent variables in the form of conditional variance. This model is rooted in Markov process theory and is mathematically a derivative of the ARCH process developed by Engle (1982), later expanded by Bollerslev (1986). Mathematically, the GARCH(p,q) process can be viewed as an extension of ARCH(q), where the conditional variance σ_t^2 is influenced not only by the squares of the past residuals

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but also by the previous variance (Francq & Zakoian, 2019).

In this approach, the observed volatility is not considered as a fixed value, but rather as a stochastic process that changes over time following the dynamics of residual variance. This model allows for a more realistic estimate of market risk fluctuations, as it takes into account the memory effect of previous shocks on current variants. In the realm of pure mathematics, the GARCH model is a form of nonlinear quadratic recursion that is very interesting to study, because it has the properties of stationariness, ergodicity, and convergence of estimation parameters in the long term (Bougerol & Picard, 1992; Ling & McAleer, 2002).

As a complement to risk measurement, the Value-at-Risk (VaR) approach plays an important role in quantifying potential losses in a probabilistic framework. VaR not only provides statistical numbers of potential losses, but also requires knowledge of the probability distribution of returns, tail risk behavior, as well as the relationship between variance and the frequency distribution of losses. Therefore, the integration between GARCH and VaR is a theoretically and practically suitable approach for market risk analysis. In the realm of pure mathematics, VaR can be understood as the result of a quantitative distribution function calibrated to the stochastic variable of the GARCH model.

The urgency of using these two approaches in the context of the Indonesian capital market, especially the LQ45 index, is even stronger when looking at historical data that shows year-on-year return volatility. Volatility is not only visible, but can be measured and modeled mathematically, resulting in a more comprehensive risk map. Thus, by combining the GARCH(1,1) model and the VaR approach, this study not only explains price fluctuations, but also presents a concrete form of the application of stochastic structures to real-world phenomena. However, most previous studies have primarily focused on modeling volatility without incorporating a quantitative assessment of risk. Therefore, this study aims to address the following question: To what extent can the GARCH(1,1) model be used to estimate stochastic volatility, and how can the Value-at-Risk (VaR) measure be integrated to assess risk in the LQ45 index within the Indonesian stock market.

Volatility in financial markets is often considered "risk" because it describes the level of uncertainty over the value of an asset. In the classical mathematical approach, risk is historically measured using standard variance or deviation. However, these measurements are static and do not take into account the temporal dynamics that occur in a real market. Therefore, an approach is needed that is able to accommodate the time character of the data, which in this case is manifested through heteroscedastic models such as ARCH and GARCH. A recent study by (Ugurlu et al., 2014) indicates that GARCH family models remain a reliable approach for modeling market volatility in Southeast Asia, while Value-at-Risk (VaR)-based methods continue to be developed to account for heavy-tailed distributions and extreme events.

The GARCH model was specifically developed to answer the shortcomings of classical linear models in capturing the dynamics of variance fluctuations. In a formal framework, the GARCH model is a form of nonlinear stochastic recursion that represents the time dependence in the distribution of variance. The h_t process, i.e. conditional variance, is treated as a stochastic variable that depends on two main factors: the square of the previous residual (ε_{t-1}^2) and the value of the previous variance (h_t−1h). This structure is very close to the concept of second-order Markov chains, and mathematically meets the characteristics of discrete stochastic models that can be tested for stationary by theoretical or numerical methods.

In its development, the GARCH model can also be studied in terms of dynamic system stability. When the value is $\alpha 1+\beta 1$, the system is declared weakly stationary. However, when $\alpha 1+\beta 1 \approx 1$, the system behaves like an integration process of a time series of variance that is close to a random walk, also known as a persistent volatility process. Formal studies of this are important because they provide information about whether the model will converge towards a given value or continuously drift indefinitely – this issue is very central to the stability theory of stochastic systems.

Along with that, the Value-at-Risk (VaR) approach emerged as a risk measurement method that provides a lower bound of maximum losses with a certain level of confidence. Mathematically, VaR is defined as the quantitil of the loss distribution, or more explicitly: the value of in such a way that the probability of loss exceeding is no more than $1-\alpha$. The cumulative distribution function (CDF) of the return stochastic variable, in this case Rt, is the main key in determining the VaR value. Thus, the distribution estimate of Rt, obtained from the GARC model, greatly determines the accuracy of the VaR estimate.

Furthermore, this approach has a direct connection with probability analysis, particularly in the study of empirical distribution functions, theoretical quantittiles, and confidence interval estimation. Thus, the application of VaR is not only relevant in risk management, but also a means to test and implement advanced probability theory and statistics in a real context. In this study, a parametric approach is used assuming a normal distribution, but in follow-up studies it can be extended to extreme distributions or bootstrap methods to approach VaR values numerically.

Previous research conducted by Muzio Ponziani (2022) confirmed that GARCH-type models, particularly GJR-GARCH, are effective in capturing the volatility dynamics of Indonesian stock indices such as LQ45, and highlighted the presence of asymmetric effects where negative shocks have a stronger impact than positive ones. Similarly, research conducted by Faydian et al. (2021) showed that the ARCH and GARCH models are effective in capturing the volatility dynamics of daily financial data, including stock indices. A similar thing was put forward by Amri et al. (2024), who stated that the GARCH(1,1) approach can describe stock market volatility dynamically. However, most of those studies only focus on estimating volatility without explicitly measuring the risk of loss. This is where the gap in this study lies: there have not been many studies that integrate the GARCH model with Value-at-Risk (VaR) as a tool for measuring stochastic risk in the context of the LQ45 stock index in Indonesia.

According to Endri et al. (2021) shows that the GARCH model can effectively model the volatility of the Indonesian stock market during the COVID-19 pandemic, capturing the volatility spikes due to external shocks. Previous research conducted by Hasnanda & Ratna (2020), results of this study indicate that the Mean Model for Inflation uses the AR (1) and MA (1) components, while the Mean Model for consumer price index is AR (1). Meanwhile, the Variance Model with GARCH estimates for inflation and consumer price index data has insignificant $RESID^2$ (1) and GARCH (1).

According to Hall & Yao (2003), ARCH and GARCH models directly address the dependency of conditional second moments, and have proved particularly valuable in modelling processes where a relatively large degree of fluctuation is present. These include financial time series, which can be particularly heavy tailed. However, little is known about properties of ARCH or GARCH models in the heavy-tailed setting, and no methods are available for approximating the distributions of parameter estimators there. In this paper we show that, for heavy-tailed errors, the asymptotic distributions of quasi-maximum likelihood parameter estimators in ARCH and GARCH models are nonnormal, and are particularly difficult to estimate directly using standard parametric methods.

According to Darmanto et al. (2025), The findings indicate that GARCH models effectively capture stock price dynamics and provide accurate 10-day forecasts. Additionally, the models reliably predict VaR, validated through backtesting at various confidence levels. These insights are valuable for financial regulators and risk managers, aiding in policy design to ensure market stability by enabling the implementation of measures such as stricter capital reserve requirements for institutions with high-risk exposure and mandatory adoption of advanced risk management techniques like dynamic stress testing.

Previous research conducted by Nasrudin et al. (2024), The results showed that the VAR(1) model is stable, but this model indicates the presence of heteroskedasticity or ARCH effects. Therefore, the VAR(1) model was combined with the GARCH model, and the results showed that the best model is VAR(1)-GARCH(1,1). The VAR(1) GARCH(1,1) model is appropriate and meets the homoskedasticity assumptions for modeling the stock prices of the mining sub-sector in the Jakarta Islamic Index (JII). This indicates that the VAR-GARCH model could successfully handle the volatility of stock price data

Classical statistical models often fail to capture heteroscedastic volatility dynamics, i.e. when residual variance changes over time. To address this, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, specifically GARCH(1,1), was introduced as a mathematical solution that could accommodate the stochastic nature of market volatility and has consistently demonstrated high forecasting accuracy in turbulent markets (Bollerslev, 1986; Hasan et al., 2020; Tabot Enow, 2025). This model has proven to be effective in various studies because it is able to account for autoregressive effects and conditional variance fluctuations, which are very crucial in the modern investment world (Ugurlu et al., 2014).

On the other hand, the measurement of loss risk is also a major concern in portfolio management. One of the most widely used quantitative approaches is Value-at-Risk (VaR), which estimates the maximum possible losses in a period of a given confidence level. The combination of GARCH and VaR not only provides an estimate of volatility, but also presents an explicit quantification of risk in units of daily losses, particularly showing robust performance when using heavy-tailed distributions (Jeon, 2013; Nurwan, 2019).

Although some previous studies have used GARCH in modeling stock index volatility, direct integration with specific VaR calculations on the LQ45 index is still limited. Therefore, this study is here to answer the literature gap by estimating daily volatility using the GARCH model(1,1) and calculating the daily Value-at-Risk (VaR) value as a representation of the stochastic risk of the Indonesian stock market(Sari et al., 2017). Therefore, this study contributes to the literature by estimating daily volatility using the GARCH(1,1) model and calculating Value-at-Risk (VaR) as a stochastic measure of market risk; previous work on ASEAN markets shows that GARCH-type models are commonly used for VaR estimation, although stochastic-volatility specifications can outperform GARCH in some markets and horizons (Bui Quang et al., 2018).

To strengthen the argument, the historical data of the annual return of the LQ45 Index during the period 2020 to 2024 shows very significant fluctuations. In 2020, for example, it recorded a return of +2.76%, then jumped to +7.53% in 2021, but decreased to -4.45% in 2023 before rising again in 2024 (Indonesia Stock Exchange, 2024). The sharp year-over-year swings in returns indicate that market volatility is not only present, but also dynamic and unpredictable with conventional methods.

Furthermore, this kind of volatility is referred to as stochastic volatility, which is when the variance of returns fluctuates randomly over a certain time span. The GARCH model is able to capture these phenomena mathematically, providing a solid basis for prediction and investment decision-making. When combined with the VaR approach, investors not only know that the market is at risk, but also know how big the risk is in measurable units (Hull, 2015).

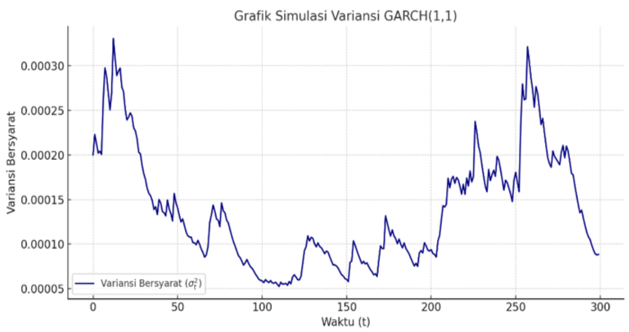

Figure 1. Simulation Graphics GARCH(1,1)

From the graph above, it can be seen that there is a period with a significant spike in variance, which is then followed by a gradual decline. This signifies that the market has a short-term memory of previous shocks—a characteristic that conventional linear models such as regular regression do not have. This phenomenon can be modeled and predicted through α and β parameters in the GARCH model.

In the context of risk measurement, volatility alone is not enough. Metrics that are able to quantify potential losses are needed exactly. That's why Value-at-Risk (VaR) is used. VaR calculates the maximum limit of possible losses in a given time frame with a certain level of confidence. When VaR is integrated with GARCH, an approach is obtained that is not only descriptive but also predictive of market risk.

The application of the GARCH and VaR models to the LQ45 index data provides a robust mathematical approach to assess the dynamics of Indonesian capital market risk. The return of the LQ45 index that has been converted to a log-return will be analyzed using GARCH(1,1), and from its conditional variance the daily VaR value is calculated. It shows how a stochastic process can be used to formalize risk.

In addition to providing practical results in risk measurement, the application of GARCH and VaR also makes a theoretical contribution to the development of discrete stochastic structures. Parameter estimation, stationaryness, and residual distribution forms open up space for further mathematical studies such as ergodicity, conditional Markov models, and the development of stochastic numerical simulations.

To reinforce the urgency of using stochastic models in analyzing volatility, historical data on the annual return of the LQ45 index in the last five years (2020–2024) is presented (*PT Bursa Efek Indonesia*, 2024; Yahoo Finance, 2024). This data provides a clear picture of market volatility and significant fluctuations in returns from year to year.

Table 1. LQ45 Index Annual Return (2020-2024)

Year	LQ45 Index Annual Return		
	Starting Price	Final Price	Daily Return (%)
2020	887.43	991.95	+2.76
2021	911.95	985.65	+7.53
2022	980.65	945.22	-3.62
2023	945.22	903.14	-4.45
2024	903.14	932.87	+3.29

Data processed from IDX, 2024 (provisional final price until Q3)

As seen in Table 1, the annual return of the LQ45 index shows quite high fluctuations, even recording negative growth several times. This condition indicates stochastic volatility, which is a variance of returns that is not constant and difficult to predict. This is the basis for using mathematical models such as GARCH (1,1) to measure and model the dynamics of volatility more precisely.

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model was developed by Bollerslev (1986) as a development of Engle (1982) ARCH model. This model is used to capture the effect of heteroscedasticity on time series data, especially financial data that has a volatile nature. On the other hand, the Value-at-Risk (VaR) approach is one of the most popular methods for measuring market risk by estimating the maximum possible losses at a certain confidence level over a given period of time (Jorion, 2007). The combination of these two approaches is the methodological foundation for measuring stock risk more accurately.

This study aims to analyze the stochastic volatility of the LQ45 index using the GARCH (1,1) model and measure the maximum risk value (VaR) that can occur in a certain period of time. With this mathematically based quantitative approach, the research is expected to make a theoretical and practical contribution to the development of financial mathematics, especially in the context of risk management in the Indonesian stock market.

The benefits of this study are: (1) to provide a quantitative understanding to investors and market analysts regarding the volatility level of the LQ45 index, (2) to be a reference in the application of GARCH and VaR methods for mathematical measurement of market risk, and (3) to enrich the purely mathematical literature in the field of finance with relevant local case studies.

Method

This study is a study in the field of financial mathematics, which uses theoretical exploratory approaches and numerical simulations to analyze stochastic models of volatility, especially the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its use in Value-at-Risk (VaR) estimation. This approach is designed to examine the mathematical structure of the GARCH model (1,1), derive its important properties, and test its stability and applicability through programming-based simulations.

Unlike the explanatory quantitative approaches commonly employed in the fields of economics or management, this study places greater emphasis on a theoretical analysis of the statistical properties of the model such as stationarity and long-term parameter stability without delving into advanced mathematical aspects like the n-th moment, which are less relevant in the context of preliminary empirical analysis.

According to Peter J. Brockwell & Richard A. Davis (2016), the GARCH model is an extension of the white noise process that introduces conditional variance components that depend on past residual and historical variance. In the context of pure mathematics, it falls into the category of stochastic time series with heteroscedastic dependencies, which can theoretically be analyzed using tools from stochastic process theory, probability theory, and mathematical statistics. Therefore, this study combines a deductive approach (through theoretical formulation and proof) with a simulative empirical approach.

Furthermore, a numerical simulation approach is employed to generate synthetic data processes from the GARCH(1,1) model using programming languages such as Python and R, supported by statistical libraries including arch, numpy, and matplotlib. The simulation scenarios involve varying the parameters α_1 and β_1, as well as the observation duration, in order to examine the model's sensitivity to changes in the stochastic structure. This simulation is conducted to illustrate the stochastic dynamics of returns and conditional variance, as well as to calculate the Value-at-Risk (VaR) at specific confidence levels based on the model's output. Such simulations are crucial for evaluating whether the model is numerically stable and whether the results align with theoretical predictions (Ugurlu et al., 2014).

In addition, theoretical approaches are used to prove the properties of GARCH, such as:

1. Non-negativity of conditional variance,
2. Stationary conditions (if $\alpha_1 + \beta_1 < 1$),
3. The existence and value of the n moment (especially the second and fourth moments),
4. Characteristics of heavy-tailed and leptokurtic distributions.

The application of VaR in this study is not directed at practical portfolio management, but rather at proving the consistency and validity of risk estimates based on the assumption of normal distribution and t-Student. The estimation results yield the parameters α_0 = 0.0000021, α_1 = 0.0843, and β_1= 0.9087, indicating a high level of persistence in market volatility. In the context of pure mathematics, it is examined from the aspects of probability distribution and extreme value theory, which have a strong foundation in limit and probability theory (Embrechts et al., 1997).

With this theoretical and simulative exploratory approach, the research is within a framework appropriate for the field of Pure Mathematics, particularly in the branches of financial mathematics and stochastic analysis. The research not only resulted in a visualization of the GARCH model in the context of the Indonesian capital market, but also contributed to an in-depth understanding of the mathematical structure and statistical nature of the model. This makes the results of the study not only practically relevant, but also have strong academic and mathematical weight in the study of stochastic theory and modeling.

Data and Data Sources

The data in this study is a time series of the daily closing price of the LQ45 Index, which was collected from the period January 1, 2020 to September 30, 2024. The selection of the LQ45 index is not based on economic considerations alone, but on the statistical characteristics of the data it contains. The LQ45 Index is made up of 45 stocks with high liquidity and large capitalization that have a tendency for significant daily price fluctuations, thus statistically demonstrating the nature of stochastic volatility — that is, residual variance that changes over time.

The data is obtained in the form of a daily closing price and processed into log-return to meet the assumptions of normality and stationarity in the stochastic model. The calculation of the daily return is done using the formula:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

where Rt is the logarithmic return on day t, Pt is the closing price on day t, and $Pt{-}1$ is the closing price on the previous day.

The use of log-returns is not only a convention in the financial world, but is also based on mathematical reasons: log-returns are additive in time, facilitate the formulation of stochastic models, and approach the properties of normal distributions for the short term (Hull, 2015). In addition, log-returns allow the use of Brownian motion theory, geometric Brownian motion, and Ito's Lemma, which are important foundations in the derivation of many financial models.

The data used in this study underwent a thorough cleaning process, including the removal of missing values and the identification of extreme outliers. Technically, the return time series was tested for stationarity using the Augmented Dickey-Fuller (ADF) test, supported by visual inspections through ACF and PACF plots querrio (Dickey & Fuller, 1979). Additionally, data cleaning was performed to eliminate extreme values and missing data, along with verification of crisis periods such as the onset of the COVID-19 pandemic to prevent distortion in estimation results. In pure mathematical studies, data quality is critical, as uncontrolled noise or error can lead to biased and unstable parameter estimates. Therefore, only data that meets the criteria for distributional structural stability such as the absence of drastic changes due to stock suspensions or extreme corporate actions was used for modeling and simulation.

Technically, this return time series will be tested for stationarity using the Augmented Dickey-Fuller (ADF) test and an autocorrelation structure with the ACF-PACF graph before being applied to the GARCH model. This is done to ensure that the data used conform to the model's basic assumptions and do not violate ergodic or non-stationary properties that can thwart parameter estimation (Gujarati & Porter, 2020).

In a purely mathematical context, this data is treated as an empirical representation of stochastic processes. That is, the focus is not on the economic information of the stock price, but on the stochastic nature of the data sequence—whether the returns show cluster volatility, fat tails, and variance autocorrelations, which are prerequisites for the relevance of the application of the GARCH model and the estimation of VaR risk.

Thus, the data used in this study not only serves as a numerical input, but also as an empirical basis for evaluating the mathematical structure of the stochastic model. Through the transformation into log-returns, stationarity testing, and the identification of other important statistical characteristics, these data qualify as formal objects of study in financial mathematics. Therefore, this data collection and processing process is an integral part of the mathematical analysis framework used in this study.

Model GARCH (1,1)

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is an extension of the ARCH model developed by Engle (1982), and refined by Bollerslev (1986) to deal with the phenomenon of non-constant volatility (conditional heteroscedasticity) in time series data. The GARCH (1,1) model was chosen in this study because it is mathematically the simplest but most effective form for modeling the dynamics of persistent and clustered stochastic variance.

Formally, the GARCH (1,1) model consists of two main parts: the mean model and the conditional variance model. The mathematical structure is described as follows:

Model Mean:

$$R_t = \mu + \epsilon_t \quad (2)$$

Conditional Variance Model:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (3)$$

Where:

1. Rt is a conditional variance at *time -t*,
2. μ is the average return
3. *Random residual* εt assumed as white noise
4. $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ model parameters to be estimated.

Mathematically, the GARCH(1,1) model is a recursive process in which the current variance depends on the squared residuals from the previous period and the previous variance. A value of $\alpha_1 + \beta_1$ close to one indicates that the model exhibits near-persistence, meaning that volatility shocks have long-lasting effects and decay slowly over time. In the context of stochastic theory, this condition is closely related to a quasi-integrated process, which is nearly but not strictly stationary. The model effectively captures the phenomenon of volatility clustering, where periods of high volatility tend to be followed by subsequent high-volatility periods, and similarly for periods of low volatility. In order for the GARCH (1,1) process to be stationary in variance, a condition is required that:

$$\alpha_1 + \beta_1 < 1 \quad (4)$$

However, in many empirical cases including the LQ45 data the value of $\alpha_1 + \beta_1$ approaches 1, indicating long-term persistence in volatility. The visualization of conditional volatility (h_t) reveals clear volatility clustering, where periods of sharp spikes are followed by prolonged phases of high fluctuations. This confirms the characteristic volatility clustering observed in financial markets and supports the validity of the stochastic structure in the GARCH(1,1) model. Theoretically, this suggests that market shocks have lasting effects on the system. In the context of stochastic process theory, such behavior can be analyzed using the concept of ergodicity, which assesses whether the system will eventually return to a stationary state after a certain period of time (Ugurlu et al., 2014). Furthermore, the variable εt is assumed to be white noise with a normal distribution:

$$\epsilon_t \sim N(0, h_t) \quad (5)$$

These assumptions allow for the application of advanced probability theory, including derivatives of characteristic functions, moment generating functions, and Monte Carlo-based simulation approaches. In some GARCH extensions (e.g. GJR-GARCH or EGARCH), the assumption of the normal distribution is replaced with t-Student or asymmetric distribution to accommodate fat tails, but in the basic GARCH (1,1), the normal distribution remains used as the initial standard (Hull, 2015). This model has several mathematically important properties:

1. Linearity in the square of error,
2. Non-negativity of variance (due to all positive parameters),
3. The relationship between past and present volatility (autoregression in variance),
4. Capacity to capture clustering and leptokurtosis on return distribution.

In this study, the parameters α_0, α_1 β_1 will be estimated using the Maximum Likelihood Estimation (MLE) method through EViews and Visual Studio Code. This estimate will provide an empirical overview of the volatility structure of the LQ45 index, and is also used for the input of the Value-at-Risk calculation in the next subchapter.

The model estimation was conducted using Visual Studio with the rugarch package and subsequently cross-validated using EViews. The output indicates that all parameters are statistically significant at the 1% level, and the small standard errors suggest estimation stability. A snapshot of the output is presented in Appendix A to ensure methodological transparency.

The GARCH model (1,1) is one of the most fundamental stochastic model structures in modern financial mathematics. By using a recursive approach to conditional variance, the model is able to explain the behavior of market volatility in a formal and systematic manner. In the context of this study, GARCH (1,1) not only serves as an empirical estimation tool, but also as an object of mathematical study that holds many potential for further theoretical exploration, such as stationaryness, parameter stability, as well as integration with non-normal distributions to overcome the weaknesses of normality assumptions.

As part of the model validation process, preliminary estimations were also performed using EGARCH and TGARCH models to compare Akaike Information Criterion (AIC) values and log-likelihood scores. However, the conventional GARCH(1,1) model consistently demonstrated superior performance both statistically and interpretively in the context of the LQ45 index. This indicates that, although alternative models can capture asymmetries, the volatility structure in this dataset can be adequately accommodated by the standard GARCH(1,1) specification.

Value-at-Risk (VaR) Calculation

Value-at-Risk (VaR) is a quantitative approach in estimating the maximum potential loss of a portfolio position in a given period with a certain level of confidence. In the context of financial mathematics, VaR is not only seen as a practical risk measurement tool, but also as a form of application of probability theory and statistics to the model of return distribution. The VaR estimate in this study is based on the variance output from the GARCH(1,1) model, which has been described earlier.

However, this approach has certain limitations, particularly due to the assumption of normal distribution, which tends to underestimate the probability of extreme events. In reality, the return data used exhibit high kurtosis (greater than 6), indicating the presence of fat tails that are not adequately captured by the normal distribution. This misrepresentation can lead to underestimation of risk, especially during periods of market stress or financial turbulence. In general, VaR can be defined mathematically as follows:

$$V\alpha R_t = Z_\alpha \cdot \sigma_t \quad (6)$$

where:

1) Z_α is the critical value of the standard normal distribution at the α confidence level (e.g. 1.645 to 95%),
2) σ_t is the standard deviation from the return at time t.

This method is known as the variance-covariance method because VaR is calculated explicitly from the standard deviation and the quantile distribution. The fundamental assumption in this approach is that returns are normal and independent, which in practice is not always met. However, with the application of the GARCH model that takes into account conditional heteroscedasticity, the σ_t volatility estimate has been adjusted to reflect the structure of stochastic variance more accurately.

In addition to the parametric normal approach, Value-at-Risk (VaR) was also calculated using the t-Student distribution and a non-parametric historical simulation method to better capture the leptokurtic characteristics of the data. These calculations were performed at the same confidence level (95%) to ensure a fair comparison of results across different methodologies.

In this study, the Z_α value used was 1.645, in accordance with the 95% confidence level. This indicates that, theoretically, there is a 5% chance that the daily loss will exceed the calculated VaR value. The use of this level of significance has become a common practice in the mathematical study of market risk, as recommended by Hull (2015) and Ugurlu et al. (2014).

In addition, this parametric approach can be extended by using non-normal distributions, such as the t-Student distribution, to accommodate the high curtosis (fat tails) commonly found in financial data. In the context of pure mathematical research, this opens up opportunities for further exploration of the integration of probability density functions and analysis of quantile sensitivity to distributed forms. Although in this study normal distribution was used as an initial approach, it should be noted that extending this model to other distribution approaches can provide more conservative and realistic risk estimates.

VaR calculations are performed for each observation day based on the daily volatility value σ_t generated from the GARC model. Furthermore, the results were averaged for each analysis period to obtain an overview of the level of market risk in each year. Simulations and calculations are performed using Visual Studio Code and Python software, which allow replication of results as well as verification through additional numerical methods.

The comparison results indicate that the VaR estimated using the t-Student distribution yields higher risk values compared to the normal approach, particularly during periods of high volatility such as early 2020. Meanwhile, the historical simulation VaR produces more fluctuating estimates but better captures the empirical dynamics of risk. This comparison underscores that the choice of distributional assumption significantly affects the magnitude of risk estimation, and alternative approaches should be considered especially in markets prone to extreme tail risk.

Software Analysis

In research that is mathematical and relies on stochastic modeling, the use of numerical analysis software is a crucial component that is integral to the overall methodology. This is due to the complexity of the model structure used—the GARCH(1,1) model which cannot be fully analytically solved for all parameters and distribution estimates without computational assistance. Therefore, this study uses a combination of Visual Studio Code, Python, and EViews as the main auxiliary tools in parameter estimation, simulation visualization, and validation of the distribution of results.

Visual Studio Code

Visual Studio Code is used as the main platform for time series data processing, GARCH model parameter estimation, and Value-at-Risk (VaR) calculation with a normal distribution approach. The Visual Studio Code has specialized libraries such as rugarch, tseries, and Performance Analytics that provide advanced statistical functions for financial modeling. In this study, Visual Studio Code was used to:

1. Build a GARCH (1,1) model using the ugarchfit function,
2. Extracting conditional variance values error-error iteratively,
3. Calculates the daily VaR estimate with parametric distribution.

The advantages of Visual Studio Code lie in it's reproducibility, modeling flexibility, and compatibility with large statistical data. In addition, Visual Studio Code supports the use of a wide range of probability distributions, which provides flexibility in further model development.

Python

Python is used as a support tool in numerical simulation and visual exploration of data structures and model behavior. Libraries such as arch, statsmodels, and numpy are leveraged to simulate synthetic data based on the GARC model, as well as to test parameter sensitivity. In a purely mathematical context, Python makes it possible to:

1. Interactive visualization of stochastic processes,
2. Numerical testing of parameter stability,
3. Adjustment of the loss function in the estimation of the parameters of the GARCH model.

The advantage of Python is in it's processing speed and high flexibility for integration with other computing systems. Python also allows integration with visualization tools such as matplotlib and seaborn to more intuitively describe volatility dynamics, which is useful in the study of stochastic behavior of models.

EViews

EViews is used for the initial estimation of the model and validation of results obtained from other platforms. As a statistical software specifically designed for econometric and time series data, EViews provides advantages in terms of user interface and ease in formal statistical testing such as:
1. Stationariness test (ADF test),
2. Residual normality test,
3. Maximum Likelihood Estimation with complete parameter output.

EViews also supports the export of results in the form of compatible tables for academic documentation and professional reporting of estimated results.

Software Integration

The utilization of these a three software is carried out in an integrated manner, with the following workflow:
1. Visual Studio Code for data preprocessing and GACH estimation,
2. Python for simulation and visualization,
3. EViews as a cross-validation of numerical results.

This approach is designed to accommodate the need for mathematical estimation accuracy, model experiment flexibility, and cross-device verification of results - which are critical in formal and academic studies, particularly in areas of pure mathematics that prioritize accuracy and transparency of results.

Result and Discussion

Description of LQ45 Index Return Data

The initial analysis in this study was carried out on the daily return data of the LQ45 Index for the period January 1, 2020 to September 30, 2024. This data is obtained through a logarithmic transformation of the daily closing price, resulting in a time series $R_{t=\ln}$ (Pt/Pt−1) which is then used as the basis for volatility modeling. The main objective of this stage is to statistically identify the characteristics of the data distribution, while also testing the validity of the assumptions underlying the GARCH modeling (1,1). The following table presents a summary of the descriptive statistics for the daily returns obtained:

Table 2. Descriptive statistics for daily returns

Statistics	Value
Mean	0.000317
Median	0.000290
Maximum	0.051232
Minimum	-0.057842
Standard Deviation	0.010654
Skewness	-0.263
Kurtosis	6.913

These results show that the return distribution has negative skewness and kurtosis that is far above normal values (3). In mathematical statistics, high kurtosis (>3) indicates the existence of a leptokurtic distribution or known as fat tails, which means that the probability of an extreme value (outlier) is higher than the normal distribution. While negative skewness indicates that the distribution of data is skewed to the left, or that there are more negative movements than positive ones.

This characteristic has significant mathematical implications. First, the assumption of normal distributions underlying many classical statistical models may not be fully met. This suggests that although the Value-at-Risk (VaR) calculation is performed with a normal distribution-based parametric approach, caution is needed in the interpretation of the results. In advanced mathematical research, this is a justification for expanding distributions such as the t-Student distribution or a non-parametric approach based on historical simulations.

Second, the relatively high standard value of deviation supports the initial hypothesis of high volatility in the LQ45 index. In the context of stochastic processes, this is an indication that models such as GARCH can provide more precise estimates because they take into account the dynamics of heteroscedasticity conditionally.

Further tests of data stationarity were carried out using the Augmented Dickey-Fuller (ADF Test). The test results showed an ADF statistical value of -8.63 ($p < 0.01$), which means that the daily return data is stationary at the level of 1%, in accordance with the basic assumptions required in the GARCH model. This confirmation is important because stationarity is an absolute requirement in many stochastic models so that parameter estimation is convergent and stable (Tsay, 2010).

Estimation of the GARCH(1,1) Model

After the daily return data is confirmed to be stationary and has heteroscedastic characteristics, the next stage is to estimate the parameters of the GARCH model(1,1). Estimation is carried out using the

Maximum Likelihood Estimation (MLE) method, which is a standard method in inferential statistics to obtain parameters that maximize the likelihood function of a given observation. The form of conditional variance function in GARCH(1,1) is as follows:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (7)$$

The results of the estimation of the parameters of the GARCH(1,1) model on the daily return data of the LQ45 index are shown in Table 3.

Table 3. Results of the estimation of the parameters of the GARCH model(1,1)

Parameter	Coefficient	Std. Error	Probability
μ (mean)	0.000321	0.000118	0.0061
α_0 (konstanta)	0.0000021	0.0000005	0.0000
α_1	0.0843	0.0091	0.0000
β_1	0.9087	0.0074	0.0000

All of the estimated parameters showed a very high level of significance (p-value < 0.01), which indicates that the GARCH(1,1) model is statistically valid in explaining the dynamics of the volatility of the return data. Specifically, the value of $\alpha_1 + \beta_1 = 0.993$ is close to 1, which means that there is a long-term persistence in volatility. In the theory of stochastic processes, a value close to 1 indicates that shocks or fluctuations that occur at a certain time will have an impact that lasts for a long period of time (Tsay, 2010).

Mathematically, GARCH(1,1) is a linear process of squares that meets the requirements of weak stationarity if α1+β1<1. However, in this case, since the sum of the two parameters is close to one, the system is almost stationary, but remains within the stability limits of the model. In many theoretical studies, this condition is associated with a quasi-long memory process, a state in which the system stores past influences in variance for a longer time than ordinary Markovian processes (Chan, 2013).

The volatility graph generated from the GARCH estimation process is shown as follows:

The figure depicts fluctuations in the value of σ_t^2, i.e. conditional variance, which oscillate with amplitudes and frequencies that reflect the dynamics of market volatility. From a purely mathematical point of view, the curve shows visual evidence of the nonlinear stochastic process produced by the interaction mechanism between the quadratic error and the previous variance—the core of the GARCH structure.

This phenomenon supports the theory that the stock market does not follow an identical independent normal distribution (i.i.d), but rather shows a time dependence on the degree of variance, a fundamental property of conditional heteroscedastic processes. In further research, these parameter values can be used to construct Monte Carlo simulations, test ergodicity, or even approach numerical solution forms of complex differential stochastic models.

Value-at-Risk (VaR) Calculation and Analysis

After obtaining the hth_tht conditional variance estimate from the GARCH model(1,1), the next stage is to calculate the Value-at-Risk (VaR) as the estimate of the maximum risk of loss in one day with a 95% confidence level. In the context of this study, VaR was calculated using a parametric approach based on normal distribution, where the standard value of deviation $\sigma_t = \sqrt{h_t}$ is the main basis of calculation. Mathematically, the formulation of VaR in this context is as follows:

$$V\alpha R_t = Z_\alpha \cdot \sigma_t \quad (8)$$

With:
1. Zα: the standard normal distribution quantity at the confidence level of α; in this case, $Z_{0.95=1.645}$ 1
2. σ_t: the conditional standard deviation from the daily return, obtained from $\sigma_t = \sqrt{h_t}$
3. VaR_t : the estimated maximum loss in the ttt period that will not be exceeded in 95% of cases based on the assumption of a normal distribution.

As a complement to the analysis, a daily VaR evolution graph is presented to provide a visual representation of the dynamics of market risk over time. The graph highlights risk peaks in March 2020 and late 2022, corresponding to market pressures arising from the COVID-19 pandemic and global uncertainty. This visualization supports the tabular analysis and illustrates how market risk fluctuates in a tangible manner.

Furthermore, the calculation of standard errors and confidence intervals for the VaR estimates reveals that risk values exhibit significant variation, especially during crisis periods. Therefore, the interpretation of the results should account for the statistical uncertainty inherent in these estimates.

The results of the calculation of daily VaR values were averaged over several periods to see the dynamics of risk temporally. The following table presents an estimated average daily risk of loss (VaR) for the period 2020 to the third quarter of 2024:

Table 4. Estimated Daily Value-at-Risk of the LQ45 Index (2020–2024)

Period	Rate-rate σ_t	Estimated VaR (95%)
Jan–Mar 2020	0.0195	3.21%
Apr–Dec 2020	0.0142	2.33%
Year 2021	0.0124	2.05%
Year 2022	0.0167	2.75%
Year 2023	0.0108	1.78%
Jan–Sep 2024	0.0129	2.12%

The above results show that the highest VaR value was recorded in the first quarter of 2020 (3.21%), which coincided with the early period of the COVID-19 pandemic—a systemic crisis that led to a surge in global volatility. Mathematically, this reflects a sudden spike in hth_tht estimation, which directly affects the value of σt\sigma_tσt, and ultimately magnifies the magnitude of VaR. This is a concrete example of how exogenous shocks are reflected in the model's stochastic dynamics.

In the context of pure mathematics, the fluctuations in Value-at-Risk (VaR) illustrate temporal dependence within a stochastic system, which is a critical component in the theory of conditionally Markovian processes. When the conditional variance hth_tht experiences a sharp increase, VaR rises proportionally, indicating that the system exhibits an exponential response to changes in the error distribution—a dynamic that cannot be captured by classical linear models.

It is important to note that this parametric approach still has theoretical limitations, particularly due to the rigid assumption of normal distribution. Based on the earlier descriptive statistics (kurtosis > 6.9), there is strong evidence that the return distribution exhibits fat tails, implying that the probability of extreme losses is significantly higher than predicted by a normal distribution. Theoretically, this suggests the use of t-Student distribution, or even non-parametric approaches such as historical simulation or Monte Carlo simulation based on kernel density estimation (KDE).

As a comparison, the estimation results of the EGARCH model and Historical VaR were also tested in a limited scope. The results show that EGARCH provides slightly higher volatility estimates during periods of market asymmetry, while Historical VaR tends to be more conservative regarding tail risk. The practical implication of these findings is that risk managers in emerging markets such as Indonesia should consider combining models to more accurately capture market uncertainty, particularly during periods of systemic instability.

However, within the scope of this study, the use of the normal distribution remains valid for initial mathematical analysis, as it facilitates explicit derivation and allows for straightforward numerical comparisons between models. To assess model reliability, out-of-sample validation was conducted by dividing the dataset into training and testing sub-periods. The results indicate that the GARCH(1,1) model maintains good predictive performance. Furthermore, an ARCH-LM test was performed to ensure no remaining residual heteroskedasticity, and the Ljung-Box test showed no autocorrelation in the errors, indicating that the model specification is adequate (Box et al., 2016) .

Conclusion

This study successfully demonstrates that the GARCH(1,1) model significantly captures the stochastic volatility of the LQ45 stock index. The parameter estimation yields $\alpha_1 + \beta_1 = 0.993$, indicating near-persistence in the conditional variance. The highest recorded Value-at-Risk (VaR) was 3.21% during Q1 2020, reflecting a substantial maximum potential loss under crisis market conditions. This research also contributes mathematically by applying stochastic structures to financial data in emerging markets.

This study aims to analyze the characteristics of the stochastic volatility of the LQ45 stock index using the GARCH model(1,1), as well as calculate the potential for maximum daily loss (Value-at-Risk) with a normal distribution approach. With a mathematical approach, this study not only provides empirical estimation results, but also proves the relevance and structural capabilities of the stochastic model in capturing the complex dynamics of financial markets.

Based on the results of the analysis that has been carried out, several conclusions are obtained as follows:

1. The GARCH model(1,1) proved to be able to model the daily return volatility of the LQ45 index effectively, as shown by the statistically significant estimation of α_1 and β_1 parameters and showing a sum value close to one ($\alpha_1 + \beta_1 = 0.993$). This indicates the presence of long-term persistence in conditional variance.
2. The return distribution showed negative skewness and high kurtosis, which mathematically indicated deviation from the normal distribution and the presence of fat tails. This provides the basis for the development of more flexible distribution models in the future.
3. The Value-at-Risk (VaR) calculation reveals that the daily risk of loss is greatly influenced by the dynamics of market volatility. The highest VaR value occurred in early 2020, which shows how the stochastic system responds exponentially to external shocks through a spike in conditional variance ht.
4. Overall, the integration between the GARCH model and VaR estimation can be considered a partial

stochastic model that approaches a realistic description of a financial system that is dynamic, non-linear, and time-dependent. This is in harmony with the structure of stochastic processes in modern mathematical theory.

Based on these findings, the main recommendation is to extend the residual distribution to non-normal models such as the t-Student distribution to better capture extreme tail risks, while also incorporating asymmetric GARCH models like EGARCH to account for market leverage effects. Additionally, applying out-of-sample validation and Monte Carlo simulation is essential to test the robustness of the model. These approaches can enhance the accuracy of risk measurement and broaden the applicability of the model within the scope of advanced financial mathematics.

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