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Vibration Control of Light Rail Vehicle Active Suspension System Using Bargaining Game-Based Distributed Model Predictive Control

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Abstract: This paper proposes a distributed model predictive control (DMPC) scheme with preview disturbance based on bargaining game theory for the active suspension control of a light rail vehicle (LRV). The objective is to improve ride comfort and safety by minimizing the deflection of suspension system and the vertical acceleration of the car bodies. The LRV is modeled as a multi-body dynamical system consisting of two power carbodies and one trailer carbody. The proposed DMPC framework decomposes the vehicle into three dynamically interacting subsystems, each governed by a local MPC controller. The optimal control inputs are determined by formulating the control problem as a bargaining game, which is then solved by a negotiation algorithm. The proposed approach is evaluated through numerical simulations under sinusoidal track disturbances and compared with a centralized MPC scheme. The simulation results demonstrate that the proposed DMPC effectively attenuates vertical acceleration and suspension deflections, thereby enhancing ride comfort and operational safety while achieving superior overall control performance. These findings indicate that the proposed DMPC provides an effective and computationally attractive alternative to centralized MPC.

Keywords: Active suspension system; Bargaining game; Distributed control scheme; Light rail vehicle; Model predictive control

Introduction

Model predictive control (MPC) is an optimization-based control approach in which a system model is used to predict future system behavior over a finite horizon. At each sampling instant, the optimal control input is obtained by solving a constrained optimization problem and applying the receding-horizon principle (Schwenzer et al., 2021). The control objective is to minimize a cost function that drives the system states or outputs toward their reference values while satisfying the prescribed constraints. Due to its ability to systematically handle constraints on both the states and control inputs, MPC has been widely applied in various fields, especially in railway transportation systems (Cavone et al., 2022; Felez et al., 2019; Jho & Ha, 2024; Luo et al., 2021; Nie & Tong, 2022; Wang et al., 2017).

Large-scale systems consist of several interacting subsystems. There are several approaches for controlling such systems, among which centralized MPC and decentralized MPC are two commonly used methods. In the centralized MPC scheme, the entire system is controlled by a single MPC controller that takes into account the interactions among all subsystems. However, this approach requires a large amount of computation and extensive information exchange, making it impractical and unsuitable for large-scale systems (Farina & Rocca, 2023). On the other hand, decentralized MPC applies a local MPC controller to each subsystem while ignoring the interactions among subsystems. Although this approach reduces computational complexity, it may result in poor performance when the interactions among subsystems are significant (Pannocchia, 2019). To address this

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limitation, distributed model predictive control (DMPC) is employed. DMPC extends the decentralized MPC framework by enabling information exchange among local controllers to coordinate their local control actions more effectively (Maxim et al., 2023). Several game theory-based approaches to DMPC have been proposed in Venkat et al. (2007) and Valencia et al. (2014). Venkat et al. (2007) developed a Feasible-Cooperation MPC (FC-MPC) approach in which the optimal control action is determined based on a Pareto solution. Meanwhile, Valencia et al. (2014) developed a DMPC framework formulated as a bargaining game, where the Nash bargaining solution is used to obtain the optimal control action.

A light rail vehicle (LRV) may be regarded as a large-scale system comprising multiple interacting subsystems. The LRV model considered in this study consists of two power car bodies and one trailer car body, based on the vehicle configuration reported by Lin et al. (2006). The connection between each power car body and its associated bogie is established through an active secondary suspension, while the bogies are coupled to the wheel axles using passive primary suspension elements. Similarly, the trailer car body is connected to its trailer bogie through a secondary suspension system, and the trailer bogie is linked to the train axle through a passive primary suspension system. The suspension system of an LRV is designed to enhance ride comfort and operational safety by reducing carbody vertical acceleration and suspension deflection while satisfying control input constraints (Wang et al., 2017).

Previous studies have investigated both passive and active suspension systems for LRVs. Lin et al. (2006) developed a passive suspension system for LRVs; however, passive suspensions cannot simultaneously achieve high ride comfort and safety because their mechanical characteristics are fixed (Omar et al., 2017). Wang et al. (2022) and Ulum et al. (2017) proposed active suspension systems for LRVs using centralized control strategies. Although these methods improve suspension performance, they require global system information and intensive computation, making them less suitable for real-time implementation in multi-car LRVs. To the best of the authors' knowledge, the application of bargaining game-based distributed MPC with preview disturbance to active suspension control of LRVs has not been reported.

Motivated by this gap, this paper proposes a distributed MPC scheme with preview disturbance for the active active suspension system of a three-car LRV. The proposed approach partitions the LRV into three dynamically coupled subsystems, where each subsystem is governed by a local MPC controller that coordinates with neighboring controllers through a Nash bargaining negotiation algorithm. Moreover, the dynamics of each subsystem are represented by a partitioned model (Valencia et al., 2014). This model describes the behavior of each subsystem by considering the effects of both local and external variables on the subsystem. Unlike existing bargaining-based DMPC methods, the proposed formulation explicitly incorporates preview disturbance into the partitioned subsystem model, allowing future track disturbances to be considered during the distributed optimization process. This feature improves the controller's ability to anticipate disturbance effects while preserving the computational advantages of distributed control. The proposed approach is important because it addresses the increasing computational and communication challenges associated with centralized control of modern multi-car railway vehicles. By combining distributed optimization, bargaining-based coordination, and disturbance preview, the proposed method provides a scalable and computationally efficient control framework capable of enhancing ride comfort and operational safety without requiring centralized computation.

Method

The methodology used to address the research problems is presented in the following diagram.

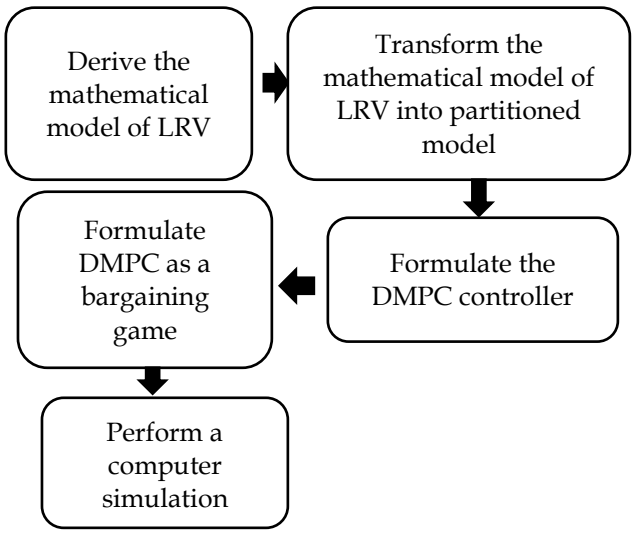


Figure 1. The flowchart of research methodology

Dynamic Equations of LRV

The LRV model considered in this study is based on the model developed by Lin et al. (2006), as shown in Figure 2. The vehicle comprises three car bodies, consisting a pair of power cars positioned at the two ends and a trailer car located between them. For each power car, the car body and its associated power bogie

are interconnected by an active secondary suspension system. The bogies, in turn, are connected to the train axles through passive primary suspension systems. Similarly, the trailer car body is coupled to its trailer bogie by a secondary suspension system, while the trailer bogie is connected to the corresponding train axles through a passive primary suspension system.

The dynamic equations governing the suspension system of a vehicle traveling along a straight track are expressed as the following system of differential equations (Ulum et al., 2017).

$$
\begin{aligned}
&m_p\ddot{y}_1 + c_1\dot{y}_1 - c_1d_1\dot{\theta}_1 - c_1\dot{y}_4 + (k_1+k)y_1 - ky_2 + (kd_2 - k_1d_1)\theta_1 + kd_3\theta_2 - k_1y_4 = f_a,\\
&m_t\ddot{y}_2 + c_2\dot{y}_2 - c_5\dot{y}_5 - ky_1 + (k_2+2k)y_2 - ky_3 - kd_2\theta_1 + kd_2\theta_3 - k_2y_5 = f_b,\\
&m_p\ddot{y}_3 + c_1\dot{y}_3 - c_1d_1\dot{\theta}_3 - c_1\dot{y}_6 + (k_1+k)y_3 - ky_2 - kd_3\theta_2 + (k_1d_1 - kd_2)\theta_3 - k_1y_6 = f_c,\\
&I_p\ddot{\theta}_1 - c_1d_1\dot{y}_1 + c_1d_1^2\dot{\theta}_1 + c_1d_1\dot{y}_4 + (kd_2 - k_1d_1)y_1 - kd_2y_2 + \left(k_1d_1^2 + kd_2^2\right)\theta_1 + kd_2d_3\theta_{2+} + k_1d_1y_4 = -d_1f_a,\\
&I_t\ddot{\theta}_2 + kd_3y_1 - kd_3y_3 + kd_2d_3\theta_1 + 2kd_3^2\theta_2 + kd_2d_3\theta_3 = 0,\\
&I_p\ddot{\theta}_3 + c_1d_1\dot{y}_3 + c_1d_1^2\dot{\theta}_3 - c_1d_1\dot{y}_6 + (k_1d_1 - kd_2)y_3 + kd_2y_2 + \left(k_1d_1^2 + kd_2^2\right)\theta_3 + kd_2d_3\theta_{2+} - k_1d_1y_6 = d_1f_c,\\
&m_{pb}\ddot{y}_4 - c_1\dot{y}_1 + c_1d_1\dot{\theta}_1 + (c_1+c_3)\dot{y}_4 - k_1y_1 + k_1d_1\theta_1 + (k_1+k_3)y_4 = k_3y_7 + c_3\dot{y}_7 - f_a,\\
&m_{tb}\ddot{y}_5 - c_2\dot{y}_2 + (c_2+c_4)\dot{y}_5 - k_2y_2 + (k_2+k_4)y_5 = k_4y_8 + c_4\dot{y}_8 - f_b,\\
&m_{pb}\ddot{y}_6 - c_1\dot{y}_3 + c_1d_1\dot{\theta}_3 + (c_1+c_3)\dot{y}_6 - k_1y_3 - k_1d_1\theta_3 + (k_1+k_3)y_6 = k_3y_9 + c_3\dot{y}_9 - f_c,\\
&m_{pb}\ddot{y}_6 - c_1\dot{y}_3 + c_1d_1\dot{\theta}_3 + (c_1+c_3)\dot{y}_6 - k_1y_3 - k_1d_1\theta_3 + (k_1+k_3)y_6 = k_3y_9 + c_3\dot{y}_9 - f_c
\end{aligned}
\tag{1}
$$

The variables y_1, y_2 and y_2 in Equation (1) represent the vertical motion of the center of gravity (c.g.) of the first power, trailer, and the second power carbodies, respectively. The corresponding pitch motions of these three car bodies are represented by θ_1, θ_2, and θ_3. For the bogies, y_4, y_5 and y_6 indicate their respective vertical displacements, corresponding to the first power, trailer, and second power bogies. Meanwhile, y_7, y_8, and y_9 characterize the vertical track irregularities encountered by the first power, trailer, and second power bogies, respectively. The value of the parameters in Equation (1) is adopted from (Ulum et al., 2017).

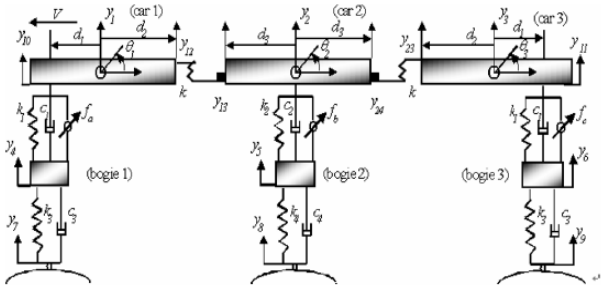


Figure 2. Multi-body dynamical model of LRVs

Partitioned Model

For the design of an LRV active suspension system based on a DMPC scheme, the overall system is split into three interacting subsystems, whose dynamics are represented by a partitioned model (Valencia et al., 2014). Subsystem 1 consists of first power-carbody and first power bogie (front part in Figure 1). Subsystem 2 consists of trailer-carbody and bogie (center part in Figure 1). Subsystem 3 consists of second power-carbody and second power bogie (rear part in Figure 1). We see that subsystem 1 interacts with subsystem 2 since subsystem 1 and 2 are connected with an articulation. It means that the variables of subsystem 2 affect the dynamic of subsystem 1. The subsystem 2 is connected to subsystem 1 and subsystem 3 using an articulation. This implies that the dynamic of subsystem 2 is affected by subsystem 1 and and subsystem 3. The dynamic of subsystem 3 is affected by subsystem 2 since subsystem 3 and subsystem 2 are connected with an articulation.

Let $x_1 = \left(y_1, \theta_1, y_4, y_7, \dot{y}_1, \dot{\theta}_1, \dot{y}_4\right)^T$, $u_1 = f_a$, and $v_1 = \dot{y}_7$ denote the state, control input, and disturbance input vectors of subsystem 1, respectively. The state, control input, and disturbance input vectors of subsystem 2 are respectively denoted by $x_2 = \left(y_2, \theta_2, y_5, y_8, \dot{y}_2, \dot{\theta}_2, \dot{y}_5\right)^T$, $u_2 = f_b$, and $v_2 = \dot{y}_8$. The state, control input, and disturbance input vectors of subsystem 3 are denoted by $x_3 = \left(y_3, \theta_3, y_6, y_9, \dot{y}_3, \dot{\theta}_3, \dot{y}_6\right)^T$, $u_3 = f_c$, and $v_3 = \dot{y}_9$, respectively. By considering the effect of local and external variables to each subsystem, the dynamic of subsystem 1, subsystem 2, and subsystem 3 are respectively given by:

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_{11}u_1(t) + E_{11}v_1(t) \tag{2}$$

$$\dot{x}_2(t)) = A_{22}x_2(t) + A_{21}x_1(t) + A_{23}x_3(t) + B_{22}u_2(t) + E_{22}v_2(t) \tag{3}$$

$$\dot{x}_3(t)) = A_{33}x_3(t) + A_{32}x_3(t) + B_{33}u_3(t) + E_{33}v_3(t) \tag{4}$$

where

$A_{11} = \begin{bmatrix} 0_{3\times4} & I_{3\times3} \\ \alpha_1 & \alpha_2 \end{bmatrix}, A_{12} = \begin{bmatrix} 0_{3\times4} & 0_{3\times3} \\ \alpha_3 & 0_{4\times3} \end{bmatrix}, B_{11} = \begin{bmatrix} 0_{3\times1} \\ \alpha_4 \end{bmatrix}, E_{11} = \begin{bmatrix} 0_{3\times1} \\ \alpha_5 \end{bmatrix}, A_{22} = \begin{bmatrix} 0_{3\times4} & I_{3\times3} \\ \alpha_6 & \alpha_7 \end{bmatrix}, A_{21} = \begin{bmatrix} 0_{3\times4} & 0_{3\times3} \\ \alpha_8 & 0_{4\times3} \end{bmatrix}, A_{23} = \begin{bmatrix} 0_{3\times4} & 0_{3\times3} \\ \alpha_9 & 0_{4\times3} \end{bmatrix}, B_{22} = \begin{bmatrix} 0_{3\times1} \\ \alpha_{10} \end{bmatrix}, E_{22} = \begin{bmatrix} 0_{3\times1} \\ \alpha_{11} \end{bmatrix}, A_{33} = \begin{bmatrix} 0_{3\times4} & I_{3\times3} \\ \alpha_{12} & \alpha_{13} \end{bmatrix}, A_{32} = \begin{bmatrix} 0_{3\times4} & 0_{3\times3} \\ \alpha_{14} & 0_{4\times3} \end{bmatrix}, B_{33} = \begin{bmatrix} 0_{3\times1} \\ \alpha_{15} \end{bmatrix}, E_{33} = \begin{bmatrix} 0_{3\times1} \\ \alpha_{16} \end{bmatrix}$

in which

$$\alpha_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{k_1+k}{m_p} & -\frac{kd_2-k_1d_1}{m_p} & \frac{k_1}{m_p} & 0 \\ -\frac{kd_2-k_1d_1}{I_p} & -\frac{k_1d_1^2+kd_2^2}{I_p} & -\frac{k_1d_1}{I_p} & 0 \\ \frac{k_1}{m_{pb}} & -\frac{k_1d_1}{m_{pb}} & -\frac{k_1+k_3}{m_{pb}} & \frac{k_3}{m_{pb}} \end{bmatrix}$$

$$\alpha_2 = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{c_1}{m_p} & \frac{c_1d_1}{m_p} & \frac{c_1}{m_p} \\ \frac{c_1d_1}{I_p} & -\frac{c_1d_1^2}{I_p} & -\frac{c_1d_1}{I_p} \\ \frac{c_1}{m_{pb}} & -\frac{c_1d_1}{m_{pb}} & -\frac{c_1+c_3}{m_{pb}} \end{bmatrix}, \alpha_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k}{m_p} & -\frac{kd_3}{m_p} & 0 & 0 \\ \frac{kd_2}{I_p} & -\frac{kd_2d_3}{I_p} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\alpha_4 = \begin{bmatrix} 0 \\ \frac{1}{m_p} \\ -\frac{d_1}{I_p} \\ -\frac{1}{m_{pb}} \end{bmatrix}, \alpha_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{c_3}{m_{pb}} \end{bmatrix}$$

$$\alpha_6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{k_2+2k}{m_t} & 0 & \frac{k_2}{m_t} & 0 \\ 0 & -\frac{2kd_3^2}{I_t} & 0 & 0 \\ \frac{k_2}{m_{tb}} & 0 & -\frac{k_2+k_4}{m_{tb}} & \frac{k_4}{m_{tb}} \end{bmatrix}$$

$$\alpha_7 = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{c_2}{m_t} & 0 & \frac{c_2}{m_t} \\ 0 & 0 & 0 \\ \frac{c_2}{m_{tb}} & 0 & -\frac{c_2+c_4}{m_{tb}} \end{bmatrix}, \alpha_8 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k}{m_t} & \frac{kd_2}{m_t} & 0 & 0 \\ -\frac{kd_3}{I_t} & -\frac{kd_2d_3}{I_t} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\alpha_9 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k}{m_t} & -\frac{kd_2}{m_t} & 0 & 0 \\ \frac{kd_3}{I_t} & -\frac{kd_2d_3}{I_t} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \alpha_{10} = \begin{bmatrix} 0 \\ \frac{1}{m_t} \\ 0 \\ -\frac{1}{m_{tb}} \end{bmatrix}, \alpha_{11} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{c_4}{m_{tb}} \end{bmatrix}$$

$$\alpha_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{k_1+k}{m_p} & -\frac{k_1d_1-kd_2}{m_p} & \frac{k_1}{m_p} & 0 \\ -\frac{k_1d_1-kd_2}{I_p} & -\frac{k_1d_1^2+kd_2^2}{I_p} & \frac{k_1d_1}{I_p} & 0 \\ \frac{k_1}{m_{pb}} & \frac{k_1d_1}{m_{pb}} & -\frac{k_1+k_3}{m_{pb}} & \frac{k_3}{m_{pb}} \end{bmatrix}$$

$$\alpha_{13} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{c_1}{m_p} & -\frac{c_1d_1}{m_p} & \frac{c_1}{m_p} \\ -\frac{c_1d_1}{I_p} & -\frac{c_1d_1^2}{I_p} & \frac{c_1d_1}{I_p} \\ \frac{c_1}{m_{pb}} & \frac{c_1d_1}{m_{pb}} & -\frac{c_1+c_3}{m_{pb}} \end{bmatrix}$$

$$\alpha_{14} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k}{m_p} & \frac{kd_3}{m_p} & 0 & 0 \\ -\frac{kd_2}{I_p} & -\frac{kd_2d_3}{I_p} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \alpha_{15} = \begin{bmatrix} 0 \\ \frac{1}{m_p} \\ \frac{d_1}{I_p} \\ -\frac{1}{m_{pb}} \end{bmatrix}, \alpha_{16} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{c_3}{m_{pb}} \end{bmatrix}$$

t denotes continouse time, and $0_{p\times q}$ and $I_{r\times r}$ refer to a zero matrice of order $p \times q$ and an identity matrice of order $r \times r$. The term $A_{12}x_2(t)$ in (2) indicates the effect of subsystem 2 to subsystem 1. Similarly, the term $A_{21}x_1(t)$ and $A_{23}x_3(t)$ in (3) indicate the effect of subsystem 1 and subsystem 3 to subsystem 2, respectively. The effect of subsystem 2 to subsystem 3 in (4) is expressed by the term $A_{32}x_2(t)$.

The partitioned model that will be used to predict the state vector of each subsystem has discrete time state-space form. Therefore, Equation (2), (3), and (4) will be discretized with sampling time T_s using c2d command in Matlab. The discrete form of Equation (2), (3), and (4) are respectively given by:

$$x_1(k+1) = A_{11}^d x_1(k) + A_{12}^d x_2(k) + B_{11}^d u_1(k) + E_{11}^d v_1(k) \quad (5)$$

$$x_2(k+1) = A_{22}^d x_2(k) + A_{21}^d x_1(k) + A_{23}^d x_3(k) + B_{22}^d u_2(k) + E_{22}^d v_2(k) \quad (6)$$

$$x_3(k+1) = A_{33}^d x_3(k) + A_{32}^d x_2(k) + B_{33}^d u_3(k) + E_{33}^d v_3(k) \quad (7)$$

where k refers to discrete time.

Controller Design

MPC is an optimal control approach that determines the control input by minimizing a predefined cost function subject to system constraints. A mathematical model of the process is employed to predict the future state trajectory over a finite prediction horizon, while the cost function provides a quantitative measure for evaluating the performance of the candidate control actions. Consider the dynamic of subsystem 1, subsystem 2, and subsystem 3 given by (5)-(7). For each subsystem i, i = 1,2,3, define the predicted state, control input, and disturbance input vectors at time step $k + j$ based on the information available at time step k by $x_i(k + j|k)$, $u_i(k + j|k)$, and $v_i(k + j|k)$, in which $x_i(k|k) = x_i(k)$. Then the prediction of state x_i, i = 1,2,3, over a finite horizon N can be expressed by:

$$X_1(k) = \Lambda_{11}x_1(k) + \Lambda_{12}x_2(k) + \Psi_{11}U_1(k) + \Phi_{11}V_1(k) + \Upsilon_{12}X_2(k) \quad (8)$$

$$X_2(k) = \Lambda_{22}x_2(k) + \Lambda_{21}x_1(k) + \Lambda_{23}x_3(k) + \Psi_{22}U_2(k) + \Phi_{22}V_2(k) \quad (9)$$

$$+\Upsilon_{21}X_1(k) + \Upsilon_{23}X_3(k)$$
$$X_3(k) = \Lambda_{33}x_3(k) + \Lambda_{32}x_2(k) + \Psi_{33}U_3(k) \quad (10)$$
$$+\Phi_{33}V_3(k) + \Upsilon_{32}X_2(k)$$

where

$$X_i(k) = \left(x_i(k+1|k), x_i(k+2|k), \dots, x_i(k+N|k)\right)^T,$$
$$U_i(k) = \left(u_i(k|k), x_i(k+1|k), \dots, u_i(k+N-1|k)\right)^T,$$
$$V_i(k) = \left(v_i(k|k), v_i(k+1|k), \dots, v_i(k+N-1|k)\right)^T.$$

Equations (8)-(10) can be rewritten in the augmented matric form as follow:

$$\Upsilon X(k) = \Psi U(k) + \Lambda x(k) + \Phi V(k), \quad (11)$$

where

$$X(k) = \left(X_1(k), X_2(k), X_3(k)\right)^T,$$
$$U(k) = \left(U_1(k), U_2(k), U_3(k)\right)^T,$$
$$V(k) = \left(V_1(k), V_2(k), V_3(k)\right)^T.$$

and

$$x(k) = \left(x_1(k), x_2(k), x_3(k)\right)^T$$

By multiplying (11) to Υ^{-1}, then the prediction of subsystem i, $i = 1,2,3$, over finite horizon N can be expressed as follow:

$$X_i(k) = F_{ii}U_i(k) + G_{ii}x_i(k) + H_{ii}V_i(k) \quad (12)$$
$$+\sum_{j\neq i}\left(F_{ij}U_j(k) + G_{ij}x_j(k) + H_{ij}V_j(k)\right)$$

in which

$$\Upsilon^{-1}\Psi = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}, \Upsilon^{-1}\Lambda = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$$
$$\Upsilon^{-1}\Phi = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}.$$

We assume that that the current state $x_i(k)$ is available at time step k, while the disturbance vector $V_i(k)$ is known through preview information for all i = 1,2,3. Moreover, we consider constraints on control input vectors. Let the admissible control input of subsystem i at time step k is restricted within the interval $u_{i_{min}} \leq u_i(k) \leq u_{i_{max}}$. Then the feasible control input sequence over a finite horizon N can be written as

$$U_{i_{min}} \leq U_i(k) \leq U_{i_{max}}, \quad (13)$$

where

$$U_{i_{min}} = \left(u_{i_{min}}, u_{i_{min}}, \dots, u_{i_{min}}\right)^T$$

and

$$U_{i_{max}} = \left(u_{i_{max}}, u_{i_{max}}, \dots, u_{i_{max}}\right)^T$$

The proposed control strategy is formulated to achieve improved passenger comfort and safer vehicle operation while simultaneously reducing the energy required for control actions. Ride comfort is improved by minimizing the vertical acceleration of the carbody in subsystem i i.e. $\ddot{y}_1, i = 1,2,3$. Safety in riding can be obtained by minimizing the deflection of secondary and primary suspensions of subsystem i, i = 1,2,3. The deflections of secondary suspension of subsystem 1, subsystem 2, subsystem 3 are respectively denoted by $y_1 - d_1\theta_1 - y_4$, $y_2 - y_5$, and $y_3 - d_1\theta_3 - y_6$ and the deflections of primary suspension of subsystem 1, subsystem 2, subsystem 3 are respectively denoted by $y_4 - y_7$, $y_5 - y_8$, and $y_6 - y_9$. To minimize both acceleration and suspension deflection can be achieved by minimizing the state vector of subsystem i. Based on these considerations, the performance of each subsystem i, i = 1,2,3, over the finite prediction horizon N is quantified by the following cost function:

$$L_i\left(U_i(k), X_i(k)\right) = \sum_{j=1}^{N} x_i^T(k+j|k)Q_i x_i(k+j|k) \quad (14)$$
$$+\sum_{j=0}^{N-1} u_i^T(k+j|k)R_i u_i(k+j|k),$$

where $Q_i, R_i \geq 0$. Replacing the expression given in (12) into (14), and subsequently removing the terms that are independent of U(k), we have the following expression:

$$L_i\left(X_i(k), U_i(k)\right) = \phi_i\left(U(k); x(k); V(k)\right), \quad (15)$$

in which

$$\phi_i\left(U(k); x(k); V(k)\right) = U^T(k)Q_{uui}U(k) + 2x^T(k)Q_{xui}U(k)$$
$$+2V^T(k)Q_{vui}U(k)$$

The notation $\phi_i\left(U(k); x(k); V(k)\right)$ indicates that the function ϕ_i depends solely on U(k), while x(k) and V(k) are parameters that the values are known at time step k. To simplify the notation, $\phi_i\left(U(k); x(k); V(k)\right)$ will henceforth be denoted by $\phi_i\left(U(k)\right)$.

Following the receding-horizon principle, the control action applied for subsystem i at time instant k is obtained from the first component of the solution to the following optimization problem:

$$\min_{U(k)} \sum_{i=1}^{3} \phi_i\left(U(k)\right), \quad (16)$$
$$\text{subject to (13).}$$

The optimization (16) will be called DMPC problem. To obtain a distributed solution to (16), the DMPC problem is reformulated as a bargaining game following the framework introduced by Valencia et al. (2014).

Game-Theoretic Formulation of the DMPC Problem

A game can be formally represented by the triplet $G = \{M, \{\Omega_i\}_{i\in M}, \{\phi_i\}_{i\in M}\}$, where $M = \{1,2,\dots,m\}$ denotes the set of players, Ω_i represents the feasible strategy set of player i, and $\phi_i: \Omega_1 \times \dots \times \Omega_m \to \mathbb{R}$ denotes the payoff function associated with the i i-th player, with R representing the set of real number (Okada, 2010). For a set of players M, a bargaining game is formulated as a pair (S, b), where $S \subseteq \mathbb{R}^m$ and $b \in \text{int}(S)$ denote the set of feasible set of profit function and the disagreement point of players, respectively (Peters, 2013). It is assumed that S to be nonempty and closed set. The definition of bargaining game (S, b) is completed by utopia point defined as $\zeta_i(S) = \max\{\phi_i: (\phi_i)_{i\in M} \in S\}$ that is assumed exists for every $i \in M$. The solution to the bargaining

game (S, b) is a payoff vector $\varphi(S, b) = (\phi_1, \dots, \phi_M)$, whose elements correspond to the profits allocated to the respective players. In the absence of cooperation, the payoff of a player is determined by its corresponding disagreement point. Consequently, when none of the players agrees to cooperate, the bargaining-game solution reduces to the disagreement-point vector, that is $\varphi(S, b) = (b_1, \dots, b_M)$.

In this research, the DMPC formulation can be interpreted as a game in which each subsystem i, i = 1,2,3, is regarded as a player i. The strategy set available to player i is defined by the admissible control-input set of subsystems i, i.e. the set of all $U_i(k)$ satisfying the constraints in (13). The profit function associated with player i is represented by the subsystem cost function in (15), i.e. $\phi_i, i = 1,2,3$. Accordingly, the DMPC problem can be expressed as the following game $G_{DMPC} = (M, \{\Omega_i\}_{i\in M}, \{\phi_i\}_{i\in M})$, $M = 1,2,3$.

The formulation of a game G_{DMPC} as a bargaining game is given by Valencia et al. (2014) by extending the static bargaining game given by Nash (1950) and Peters (2013) to discrete-time dynamic bargaining game. The discrete-time dynamic bargaining game is defined by a tuple $\{(\Gamma(k), \eta(k))\}_{k=0}^{\infty}$. Here, $\Gamma(k) \subseteq \mathbb{R}^m$ denotes the admissibl set of all cost funtions $\phi_i(U(k))$ at time step k such that $U(k) \in \Omega(k)$, where $\Omega(k) = \prod_{i=1}^{m} \Omega_i(k)$, and $\eta(k) = (\eta_1(k), \eta_2(k), \dots, \eta_m(k))$ denotes a tuple of disagreement point of players where the disagreement point of player i at time step k is formulated as

$$\eta_i(k+1) = \begin{cases} \eta_i(k) + (\phi_i(U(k) - \eta_i(k)), & \eta_i(k) < \phi_i(U(k) \\ \eta_i(k) - \alpha(\eta_i(k) - \phi_i(U(k)), & \eta_i(k) > \phi_i(U(k) \end{cases} \quad (17)$$

Table 1. Negotiation model for DMPC with preview disturbance

Step	Command
1	At each sampling instant k, every subsystem broadcasts its current state $x_i(k)$, disagreement point $\eta_i(k)$, and value function $V_i(k)$ to all other subsystems.
2	Each subsystem solves the optimization problem in (18) using the available shared information.
3	If the optimization problem has a feasible solution, subsystem i applies the first element of the optimal control sequence $U_i^*(k)$. Otherwise, the subsystem uses the first element of its previously available control sequence $U_i(k)$.
4	The disagreement point is updated according to $\eta_i(k+1) = \eta_i(k) - \alpha(\eta_i(k) - \phi_i(U_i(k))$, if (18) is feasible. In the infeasible case, the disagreement point is updated as $\eta_i(k+1) = \phi_i(U(k))$.
5	Each subsystem transmits its updated control sequence together with the new disagreement point to the remaining subsystems.
6	The above procedure is repeated at the next sampling instant.

for all $i \in M$, with $0 < \alpha < 1$. Finally to complete the formulation of DMPC scheme as a discrete time dynamic bargaining game, the utopia point $\zeta(\Gamma(k))$ is defined as $\zeta(\Gamma(k)) := \min\{\phi_i(U(k)) : \phi_i(U(k)) \in \Gamma(k)\}$ exists for every $i \in M$. The bargaining solution of the game G_{DMPC} at time step k is determined in a distributed manner through the solution of the following optimization problem:

$$\max_{U_i(k)} \sum_{r=1}^{m} w_r \log\left(\eta_r(k) - \sigma_r(U_i(k) - U_{-i}(k))\right) \quad (18)$$

subject to

$U_i(k) \in \Omega_i(k)$,

$\eta_r(k) > \sigma_r(U_i(k), U_{-i}(k))$,

$\sum_{r=1}^{m} w_r = 1, w_r > 0$ for all r.

where $\sigma_r(U_i(k), U_{-i}(k)) = \phi_r(U(k))$ for $r = 1, \dots, m$, and $U_{-i}(k) = [U_1^T(k), \dots, U_{i-1}^T(k), U_{i+1}^T(k), \dots, U_m^T(k)]$.

To solve the optimization problem in (18), this paper adopts the negotiation framework originally introduced by Valencia et al. (2014). Unlike the original approach, the proposed negotiation model is modified to accommodate the distributed MPC scheme in the presence of preview disturbances. This modification is necessary because the controller developed in this study explicitly incorporates future disturbance information into the optimization process. The proposed negotiation model is summarized in the following algorithm.

Result and Discussion

To evaluate the effectiveness of the proposed control strategy, a series of numerical simulations were carried out using the parameters summarized below. The sampling period was selected as $T_s = 0.01$s with a prediction horizon of $N = 6$. The initial state and control input of each subsystem were initialized as $x_i(0) = 0$ and $u_i(0) = 0$, respectively, for $i = 1,2,3$. The weighting matrices employed in the cost function (14) were chosen as $Q_i = I_{7\times7}$ and $R_i = 10, i = 1,2,3$. Moreover, the coefficients of the convex combination in (17) were fixed as $w_1 = w_3 = \frac{1}{3}$ and $w_2 = \frac{2}{3}$. The initial disagreement points for Subsystems 1, 2, and 3 were set $\eta_1(0) = \eta_2(0) = \eta_3(0) = 80$, respectively. Finally, the admissible control input of each subsystem was constrained by $-10 \le u_i(k) \le 10, i = 1,2,3$. To investigate the disturbance rejection capability of the proposed controller, each subsystem was subjected to a

sinusoidal disturbance $v_i(k) = a\sin(\omega k), i = 1,2,3$, where the disturbance amplitude and angular frequency were selected as $a = 0.05$ m and $\omega = 7.8025$ rad/s, respectively. The resulting disturbance profile is illustrated in Figure 3.

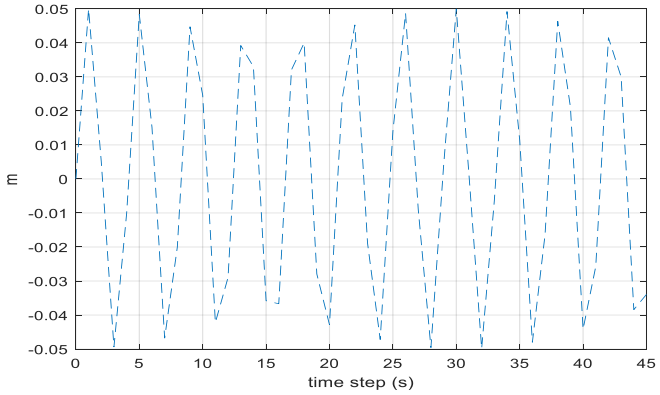


Figure 3. Sinusoidal track profile

Figure 4 compares the secondary suspension deflections obtained using the proposed DMPC and the (centralized) MPC controller over a simulation interval of 45 s. Overall, the proposed DMPC achieves superior vibration attenuation. For Subsystem 1, the secondary suspension deflection under DMPC remains close to the equilibrium position, with a maximum displacement of only 0.0037 m, whereas MPC produces a significantly larger peak displacement of 0.0172m. For Subsystem 2, both controllers exhibit comparable dynamic responses; however, DMPC still reduces the maximum suspension deflection from 0.0182m (MPC) to 0.0132m. A similar trend is observed for Subsystem 3, where the peak suspension deflection decreases from 0.0172 m under MPC to 0.0141 m under DMPC. These results demonstrate that the proposed distributed controller consistently suppresses the secondary suspension deflection more effectively than MPC, thereby contributing to improved operational safety.

The primary suspension deflections are presented in Figure 5. As can be observed, the proposed DMPC substantially outperforms MPC for all subsystems. For Subsystem 1, the maximum primary suspension deflection is limited to only 0.0014 m under DMPC, whereas MPC results in a considerably larger displacement of 0.0670 m. Likewise, for Subsystem 2, DMPC reduces the maximum deflection from 0.0548m to 0.0037 m. Similar improvements are obtained for Subsystem 3, where the peak displacement decreases dramatically from 0.0670 m under MPC to 0.0013 m under DMPC. Since both primary and secondary suspension deflections are directly related to suspension travel and mechanical safety, these results indicate that the proposed DMPC provides a significantly higher level of ride safety than MPC.

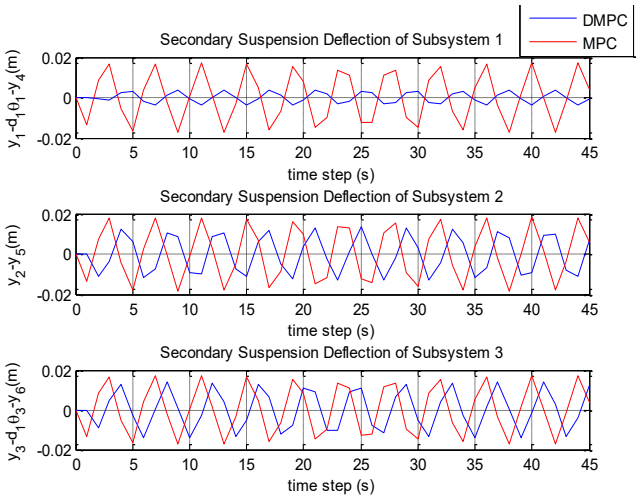


Figure 4. The deflection of secondary suspension

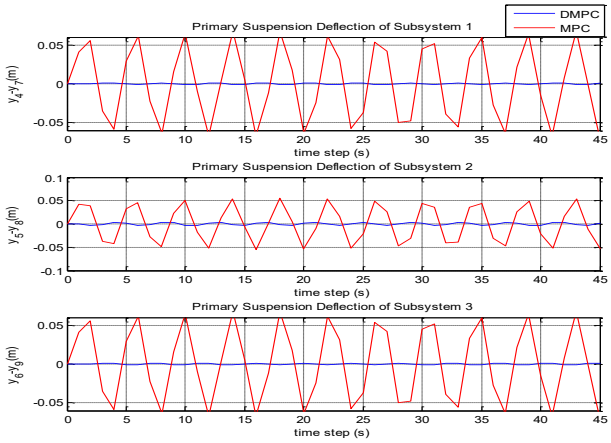


Figure 5. The deflection of primary suspension

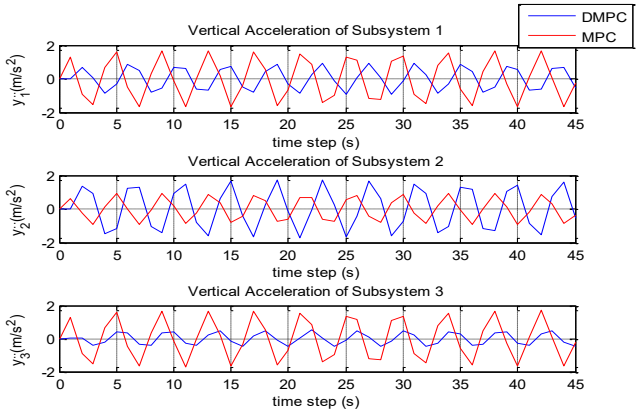


Figure 6. The vertical acceleration of system

Figure 6 illustrates the vertical acceleration responses of the three subsystems. In general, the proposed controller provides improved ride comfort by

reducing the vehicle body acceleration. For Subsystem 1, DMPC decreases the peak vertical acceleration from $1.6908\ \mathrm{m/s^2}$ (MPC) to $0.9151\ \mathrm{m/s^2}$. A similar improvement is observed for Subsystem 3, where the maximum acceleration is reduced from $1.6908\ \mathrm{m/s^2}$ to $0.5018\ \mathrm{m/s^2}$. In contrast, Subsystem 2 exhibits a different behavior. Specifically, MPC achieves a lower peak acceleration of $0.9151\ \mathrm{m/s^2}$, whereas DMPC produces a larger value of $1.7072\ \mathrm{m/s^2}$. Although MPC performs slightly better for Subsystem 2, the proposed DMPC yields lower peak accelerations for two of the three subsystems while providing substantial improvements in suspension deflections. Consequently, considering the overall vehicle performance, the proposed distributed controller offers a better compromise between ride comfort and ride safety than MPC.

Conclusion

This paper has presented a distributed model predictive control (DMPC) scheme with preview disturbance compensation for the active suspension system of a light rail vehicle (LRV). The proposed control strategy aims to improve both ride safety and passenger comfort by reducing suspension deflections and vehicle body acceleration. Under sinusoidal track disturbances, the proposed DMPC consistently reduced suspension deflections for the three subsystems compared with conventional MPC. In particular, the maximum secondary suspension deflections were reduced from 0.0172 m, 0.0182 m, and 0.0172 m under MPC to 0.0037 m, 0.0132 m, and 0.0141 m under DMPC for subsystems 1, 2, and 3, respectively. Similarly, the maximum primary suspension deflections decreased substantially from 0.0670 m, 0.0548 m, and 0.0670 m to 0.0014 m, 0.0037 m, and 0.0013 m, respectively. The proposed DMPC also reduced the maximum vertical acceleration of subsystems 1 and 3 from 1.6908 $\mathrm{m/s^2}$ to 0.9151 $\mathrm{m/s^2}$ and from 1.6908 $\mathrm{m/s^2}$ to 0.5018 $\mathrm{m/s^2}$, respectively, although subsystem 2 exhibited a higher acceleration than under MPC. Overall, the substantial reduction in suspension deflection and the improvement in vertical acceleration for most subsystems indicate that the proposed DMPC provides improved ride safety and comfort compared with (centralized) MPC. Future research should investigate the proposed DMPC framework under more realistic and diverse track disturbances, including stochastic and measured track irregularities, to further assess its robustness. The effect of communication delays, measurement uncertainties, actuator limitations, and model uncertainties should also be incorporated into the control formulation. In addition, experimental or hardware-in-the-loop validation using an LRV suspension test platform would be valuable for assessing the practical applicability of the proposed method. Further research could also investigate adaptive or learning-based bargaining mechanisms to improve the control performance of individual subsystems, particularly for cases in which the distributed controller does not provide uniform improvements in vertical acceleration across all subsystems.

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Author Contributions
Conceptualization, Z.U. and Z.; methodology, software, formal analysis, investigation, writing—original draft preparation, Z.U.; validation, resources, data curation, writing—review and editing, visualization, supervision, Z.

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Conflicts of Interest
The authors confirm that there is no conflict of interest associated with this work.

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