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Hypothesis Design of Learning Trajectory Volume Building Flat Sided Spaces in Junior High Schools: Based on Van Hiele's Theory

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Abstract: This study aimed to develop a hypothesis learning trajectory that can help students understand the concept of volume polyhedron based on van Hiele's theory. This study used a design research method with a type of validation studies to create a design framework for learning trajectories passed by students in mathematics learning. The study focusedon the first stage, i.e. preliminary design to develop a sequence of volume learning activities to polyhedron used the stages of the van hiele theory. The results of this study show the learning activities that will be traversed by students and teachers in understanding the concept of volume up the flat side space through the four stages of van Hiele i.e. 1) visualization stage at which students find the prism volume; 2) analysis phase at which students find the volume of cuboid obtained from the prism volume; 3) informal deduction phase at which student finds the volume of the cube from the building of the cuboid with the same length and height; 4) formal deduction phase at which student to find the formula for the volume of pyramid.

Keywords: Hypothetical learning trajectory; Van hiele's theory; Volume polyhedron

Introduction

Mathematics learning is essentially carried out in stages and continuously from one to another so that students understand the flow of learning carried out by the teacher and the teacher understands the flow of student learning. This is in accordance with the words of Novianti et al. (2016) that learning mathematics should be done in stages starting with understanding ideas and a simple concept can be raised to a higher level through learning. In addition, mathematics must be meaningful to help students understand concepts and apply them in real life.

The application of mathematics learning must be well planned by the teacher so that it fits the characteristics of each student taught in class because according to Wijaya (2015) in making lesson plans, individual student differences must be considered. As teachers gain insight into students' thinking and learning, they will begin to formulate conceptions about student understanding and use this knowledge to develop conjectures about the sequence of tasks and learning activities. Thus, they will formulate a hypothetical learning trajectory to plan the instruction (Amador et al., 2013).

The learning trajectory hypothesis was formulated in advance by Simon (1995) revealing that there are three parts to the hypothetical learning path, namely: indicative learning objectives, learning activities, and hypothetical learning processes which are assumptions about the development of students' thinking and understanding of learning activities. These three things from the learning trajectory hypothesis Figure 1 is intended as a representation of the situation.

The learning trajectory hypothesis or hypothetical learning trajectory (HLT) according to Rezky et al. (2018) can be compared to planning a learning path. Understanding the possible routes to our destination allows us to select the possible routes to our destination in order to choose a good route. The route selection or planning that we do is adjusted to the learning characteristics of students because students have their

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own learning characteristics. As stated by Wittek (2013) that "student trajectories of learning bring together third parties in unique ways...". Nurdin (2011) says that "The learning trajectory is made up of three components: the learning goals, the learning activities, and the hypothetical learning process". This means that there are three parts to the learning flow, namely learning objectives, learning activities and the hypothetical learning process.

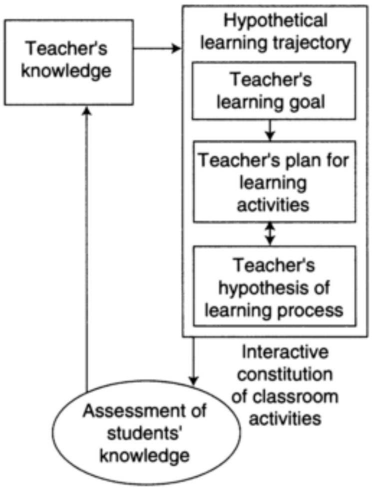

Figure 1. Mathematics learning circle (Simon, 1995)

The process of obtaining this learning flow continues to develop according to the characteristics of students learning in class. The first part is that the teacher's learning objectives are influenced by two factors, namely (1) the teacher's knowledge of mathematics, the importance of the teacher's understanding, because the teacher needs to know the continuous process of learning mathematics, and (2) the teacher's hypothesis about student knowledge. Teachers need to know the extent to which students' understanding of what is learned so that learning in class becomes optimal.

The second part of the teacher's lesson plan is influenced by four factors, namely (1) the teacher's understanding of mathematics, (2) the teacher's understanding of mathematical functions and representations, (3) the teacher's teaching theory, and (4) the teacher's guarding theory. In the third part, the teacher's hypothesis about learning is influenced by three factors, namely (1) the teacher's knowledge of mathematics, (2) the teacher's hypothesis about student knowledge, and (3) the teacher's knowledge of student learning. For more details, see the HLT compiled by Simon (Amador et al., 2013) in Figure 2.

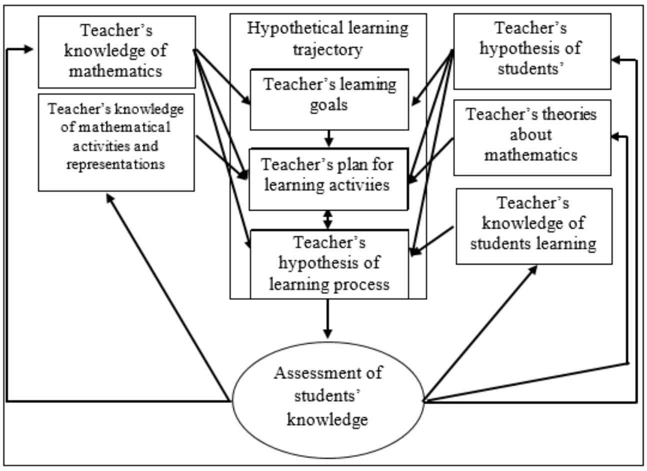

Figure 2. HLT in mathematics learning by Simon (Amador et al., 2013)

This research focuses on learning geometry with a flat side volume based on Van Hiele's theory. Van Hiele geometry learning consists of five hierarchical levels. The five levels referred to are level 0 (Visualization), level 1 (analysis), level 2 (Abstraction), level 3 (Deduction), and level 4 (Rigor) (Haviger et al., 2015)

The five levels in van Hiele's theory are stages that take place sequentially, meaning that it takes place step by step where the child goes through the initial stages to go to a higher stage than before. So to find out the level of student geometry, the instructions that need to be carried out by the teacher must be in accordance with the development of the student's mindset. This is in line with the opinion of Hiele (1999) that teaching must always follow the thoughts and behavior of students and aims to advance development from one level to another. The stages of geometric reasoning based on Van Hiele's theory are shown in Figure 3.

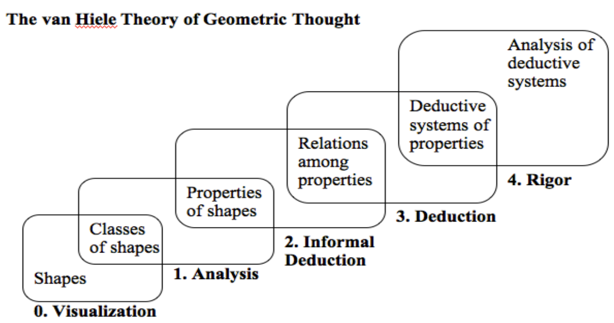

Figure 3. Stages of geometry development according to Van Hiele's Theory (Walle, 2007)

The five stages of Van Hiele's geometrical thinking levels are:

Level 1 (Recognition/Visualization)

In the first stage, students observe objects in pictures, and decisions are based more on perception than reason. And students treat this figure without characteristics, definitions, and descriptions (Rezky et al., 2018). Students know geometric shapes as a whole based on visual aspect, students do not know the characteristics of geometric shapes (Hiele, 1999). The products of level 1 thinking are classes or groups of shapes that appear "same" (Walle, 2007).

Level 2 (Analysis)

At this stage students identify the characteristics of objects, images or shapes. Where students cannot fully define and describe objects. A student analyzes shape attributes and the relationships between shape attributes. Murray notes that at level 2 terminology and symbols are meaningful to students and they can formulate their own definitions. Definitions are accepted as binding for arguments and logical discussion (Rezky et al., 2018). The product of thought at level 2 is the nature of form (Walle, 2007).

Level 3 (Informal Deduction)

At this stage, students are able to reason with meaningful descriptions. They can also logically link previously found properties/rules by providing or following informal arguments (Rezky et al., 2018). The result of level 3 reasoning is the relationship between geometric properties (Walle, 2007).

Level 4 (Formal Deduction)

At this stage, students can prepare evidence. At this stage students already understand the deductive-axiomatic thinking process (Rezky et al., 2018). The product of thought at level 4 is the deductive axiomatic system for geometry (Walle, 2007).

Level 5 (Rigor)

At this level, students' understanding has begun to understand the importance of the accuracy of the basic principles in a proof (Rezky et al., 2018). The product of thinking at level 5 is comparisons and contracts between different axiomatic systems for geometry (Walle, 2007).

The focus of this research is the four stages of van Hiele's theory, namely visualization, analysis, informal deduction and formal deduction. The learning process using van Hiele's theory has steps in each stage (Breyfogle et al., 2020). The description of the five steps is as follows.

Inquiry/information

At this stage, students get to know the domain they are working on, for example testing examples and non-examples. Teachers and students seek talks and activities about the objects studied at that level. Observations must be made, questions must be raised and the vocabulary for this level must be introduced (Crowley, 1987; Fuys et al., 1988; Hiele, 1999).

Directed orientation

At this stage, students work on tasks that involve different relationships from the problems formed. Students examine the topic of the lesson through the material that has been arranged sequentially by the teacher. The teacher guides students to look at the characteristics of the object to be studied. In this way, various materials become short tasks aimed at finding certain answers (Crowley, 1987; Fuys et al., 1988; Hiele, 1999).

Explicationn

At this stage, the teacher explains the relationship between the spatial properties and motivates students to apply them in discussions and assignments. Students are aware of the relationship between geometric terms, try to express it in their own language and learn the appropriate technical language with the material (Van (Crowley, 1987; Fuys et al., 1988; Hiele, 1999).

Free orientation

At this stage, the teacher gives questions for students to solve in different ways and broadens students' prior knowledge of geometry. Students are given more complex assignments (Crowley, 1987; Fuys et al., 1988; Hiele, 1999).

Integration

At this stage, the purpose of learning is to summarize. Students summarize what they have learned. The goal of this step is not to explore a new idea, but to try to bring together what has been explored and discussed into a logical network that is easy to describe and implement. Networks are explained in Language and Mathematical Conceptualization (Crowley, 1987; Fuys et al., 1988; Hiele, 1999).

Flat side shapes are subject matter for Junior High School (SMP), with one of the materials discussing the volumes of geometric shapes such as cubes, rods, prisms and pyramids. When the teacher teaches geometric shapes, prism shapes can be one of the basic references for obtaining the volume of other geometric shapes such as cuboids, cubes and pyramids. This is in accordance with what was revealed by Sardjana (2008) regarding the definition of a prism. When it is lowered further, we get an upright parallel epipedum whose edges are perpendicular to the base plane and the rectangular base can be called a right-angled parallel epipedum or cuboid (Sardjana, 2008).

After obtaining the cuboid, a cube can be formed from the cuboid whose long edge are the same length. This is in accordance with what was said by Sardjana (2008) that a parallel epidedum whose three edges meet at one point is the same length as a rhomboeder, and a right-angled rhomboeder is a cube. To find out more details, the formation of prisms to cubes is shown in Figure 4.

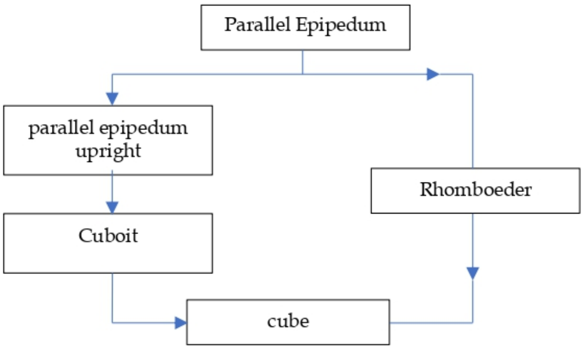

Figure 4. Flow of epipedum parallel formation (Sardjana, 2008)

A pyramid can be formed from cubes divided by six in such a way that a pyramid shape can be formed. See Figure 5 for details.

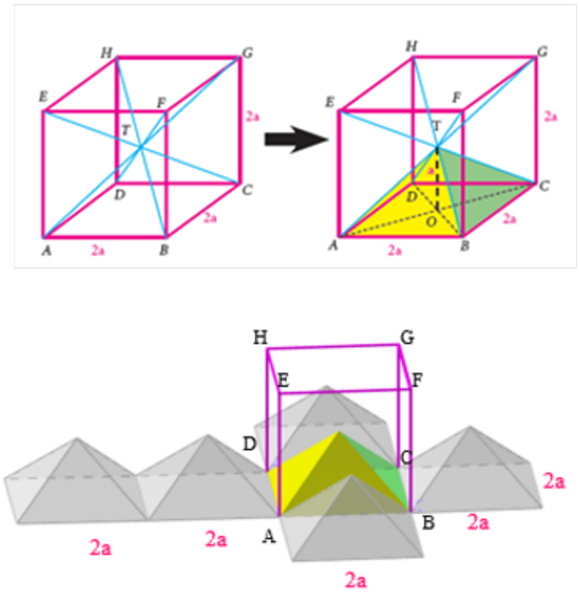

Figure 5. Formation of six pyramids from cubes

This geometry material is taken because geometry plays an important role in learning other concepts in learning mathematics (Walle, 2007). However, in its application in the field, junior high school students in Indonesia still lack the mastery of geometry material. This is proven based on the TIMSS 2011 results, out of 45 countries, Indonesia only occupies the 41st position for the mathematics abilities of Grade VIII students with an average score of 386 (Mullis et al., 2012) while in PISA 2015, out of 65 countries, Indonesia occupies the 64th position, with an average score of 386 (O.E.C.D., 2017). More specifically, the score of Indonesian students in TIMSS 2011 was 377 for the geometry domain (Mullis et al., 2012). Judging from the average math ability of junior high school students in Indonesia based on the results of the national exam (UN) in the last three years according to BSNP (2016), (2017), and (2018) the average percentage of students' math UN results in 2015/2016 was 50.24%, 2016/2017 of 54.75%, and in 2017/2018 it was 43.08%.

If we further look at geometry and measurement material in the Mathematics National Examination for junior high school students in 2018, the percentage of students who answered correctly nationally according to BSNP (2018) was only 42.80%. This shows that students' understanding of geometry and material measurement is still relatively low.

Based on the description, the research question is "How to Design Learning Trajectory Volume Build Flat Sided Spaces in Junior High Schools (SMP) Based on Van Hiele Theory".

Method

This type of research is design research. The researcher did a description and developed a hypothetical learning trajectory (HLT) for geometric material, especially the volume of prisms, cuboids, cubes and pyramids. Design research is a cyclic process carried out to develop the learning process, this is in accordance with what is expressed by Gravemeijer et al. (2003) which states that "design research is a cyclic process of designing instructional analyzing the sequences, testing and revising them in classroom learning from the class so that the cycle of settings, and then design, revision, and implementation can begin again".

Gravemeijer et al. (2009) found that "design research that aims at developing a local instruction theory". In the sense that this design research aims to develop a theory of local instruction. This design research model uses the Gravemeijer and Cobb model (Gravemeijer et al., 2009) where the ultimate goal is to produce a local instruction theory design. The research design carried out is as follows.

Nieveen, McKenny and Akker divided design research into two main sections, namely validation studies and development studies (van den Akker et al., 2006). According to Nieveen, McKenny and Akker validation studies are learning paths in the development, processing and validation of learning theories about learning and the resulting intervention results in the Lesson Plans. The resulting contributions are (a) microtheories (level of learning activities), (b) local instructional theory (level of learning process) and (c) domain specific instruction theory (level of pedagogical knowledge content).

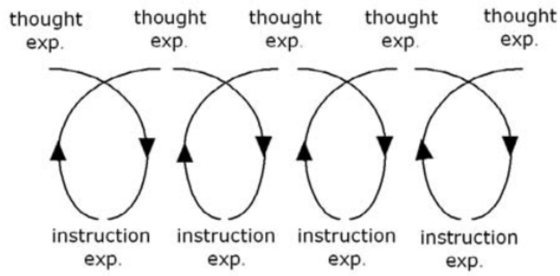

Figure 6. Design research conducted

Gravemeijer and Cobb divided research design into three main phases, namely preparing for the experiment, experimenting in the classroom and conducting retrospective analyses (van den Akker et al., 2006). The retrospective analysis phase is involved in developing the LIT and proposing additional issues or innovations. There are two important aspects of design research, namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). In general, according to Prahmana (2017), design research stages are divided into 3 stages, namely:

Phase I: Preliminary Design

According to Widjaja, the main objective of this research is to develop a set of learning activities and design instruments to evaluate the learning process (Prahmana, 2017).

Phase II: Design Experiment

In this second stage, the researcher tested the learning activities planned in the first stage. The purpose of this test is to test and predict students' strategies and thinking during actual learning. The test phase is divided into two stages, namely the (teaching experiment) and pilot experiment.

Phase III: Retrospective Analysis

After the experience of learning to plan Action. From what is obtained from learning carried out in class is evaluated retrospectively. The aim of the retrospective analysis is to develop a general theory of local education.

The focus in this research is design research with the type of validation studies. This research was conducted at SMPN 4 Baubau in Semester II. This research procedure focuses on the early stages of design research, namely preliminary design which aims to develop a sequence of learning activities and design instruments to evaluate the learning process. Data collection techniques in the preliminary design stage, namely observations made during the learning process occur with the aim of obtaining HLT data implemented in learning, which is used as input material for HLT at a later stage. The analysis was carried out using descriptive methods to describe the information obtained throughout the learning process.

Result and Discussion

This study was designed to hypothesize learning trajectories in learning volume material using the four stages of van Hiele's theory. The hypothesis is how to predict the flow of students' thinking and the actions taken by the teacher based on learning objectives that refer to the curriculum in Indonesia. The objectives of learning the volume of a flat side shape are: (1) Determine the volume of cubes, cuboids, prisms and pyramids; and (2) solving problems related to the volume of cubes, cuboids, prisms and pyramids.

For learning activities and students' thinking flow hypotheses are carried out in accordance with the 4 stages of van Hiele's theorem. In the first stage, namely the visualization level, the activities carried out by the teacher and the students' thinking flow hypothesis are shown in Table 1.

Table 1. Level 1 Visualization

Activity Details	Teacher Activity	Student Thinking Flow Hypothesis
Information	Provides information about some prism shapes.	Pay attention to the display given by the teacher about some prism shapes.
Direct orientation	Invite students to determine the elements of a prism.	Determine the elements of a prism.
Explication	The teacher asks students to explain the prism elements, namely the base and height of the prism.	Students explain the base of the prism and the height of the prism.
Free orientation	The teacher asks students to find the volume of prisms with different bases.	Students look for the volume of a prism with a different base.
Integration	The teacher asks students to deduce the formula for the volume of a prism with the various types of bases shown.	Students conclude that the volume of a prism can be found by knowing the base area of the prism which will be multiplied by the prism's height or V = Base area x prism height.

Table 2. Level 2 Analysis

Activity Details	Teacher Activity	Student Thinking Flow Hypothesis
Information	The teacher provides information about prisms with rectangular base shapes, namely: a square or a rectangle.	Students pay attention to the information provided about the shape of the prism.
Direct orientation	The teacher invites students to identify the similarity of the prism shape with the cuboid shape.	Students identify the similarity of a prism shape with a cuboid shape
Explication	The teacher asks students to explain the similarities in the shape of a rectangular prism and a cuboid.	Students explain the similarities in the shape of a rectangular prism and a cuboid.
Free orientation	The teacher asks students to recalculate the volume of a rectangular prism with a square or rectangular base.	Students calculate the volume of a prism with a square or rectangular base.
Integration	The teacher asks students to conclude the relationship between the prism volume formula and the volume of a cuboid.	Students can find the relationship between the formula for the volume of a prism and the volume of a cuboid, with the formula for the volume of a cuboid being the area of the base multiplied by the height of the figure or in general that the volume of a cuboid is the length times the width times the height or $V = l \times w \times h$.

Table 3. Level 3 Informal Deduction

Activity Details	Teacher Activity	Student Thinking Flow Hypothesis
Information	The teacher provides information about the shape of a cuboid with a square base and the height of the shape is equal to the length of the base.	Students pay attention to the information about the shape of the given cuboid.
Direct orientation	The teacher asks students to determine the similarity between cuboid and cubes as they have known so far.	Students determine the similarities that occur between cuboid and cubes.
Explication	The teacher asks students to explain the similarities between the cuboid and cubes.	Students explain the similarities between the shapes given.
Free orientation	The teacher asks students to calculate the volume of a cuboid with a square base and the height of the long figure is the same as the length of the base.	Students calculate the volume of a cuboid with a square base and the length of the base is equal to the height of the figure.
Integration	The teacher asks students to conclude what is obtained at this stage.	Students conclude that a cuboid with a rectangular base and a height equal to the length of the base is a cube. So that the volume can be found by multiplying the edge times the edge times the edge or $V = r \times r \times r = r^3$

Table 4. Level 4 Formal Deduction

Activity Details	Teacher Activity	Student Thinking Flow Hypothesis
Information	The teacher provides information about the shape of the cube in which six pyramids are formed in the same and congruent shapes.	Students pay attention to information from the teacher about pyramids formed from cubes.
Direct orientation	The teacher asks students to determine the similarities and differences that occur between cubes and pyramids seen from the length of the base and the height of the shape.	Students determine the similarities and differences between the cubes and the pyramids formed.
Explication	The teacher asks students to explain what happened after the cube was formed into six congruent pyramids.	S Students explain that the length and width of the base of the pyramid are equal to the length of the side of the cube, while the height of the pyramid is half the length of the edge of the cube.
Free orientation	The teacher asks the students to find the volume of the cube again and asks the students to determine the volume of the pyramid.	Students find the volume of the cube and determine the volume of the pyramid that the volume of the pyramid is one-sixth of the volume of the cube.
Integration	The teacher asks students to make conclusions from what has been achieved in this step.	Students are able to determine the volume of the pyramid that comes from one-sixth of the volume of a cube or the volume of the pyramid can be found by multiplying the area of the pyramid's base and the height of the pyramid, the result is divided by three in this case $V = 1/3 \times$ Base area $\times$ pyramid height.

At this stage students can identify and determine the volume of several prisms with different base shapes. So that students can conclude the formula for the volume of the prism they want to find, that the volume of the prism is the product of the area of the base and the height of the prism (V = Base area x prism height). The shape of the base is very influential; this is because the shape of the base in the form of a rectangle such as a square or rectangle forms students to the next level, namely the level of analysis. This happens because the shape of the prism base is square or rectangular, so the shape of the prism is similar to a cuboid shape. Table 2 provides more detailed information about the activities carried out by the teacher and the students' thinking flow hypothesis.

At this stage it can be seen that students have been able to analyze the relationship between rectangular prisms with square or rectangular bases and cuboid shapes. As a result, students have been able to determine the formula for the volume of a cuboid derived from the prism volume, namely the area of the base multiplied by the height. So that in general students get the formula for the volume of a cuboid, namely length times width times height (V = l x w x h).

The next stage is the level of informal deduction (informal deduction). At this stage students determine the volume of the cube, this is obtained from students' understanding of the volume of the cube. For the stages of activity and hypotheses from student learning flow can be shown in Table 3.

At this stage students have been able to distinguish the three types of shapes, namely prisms, cuboids and cubes. So that students have been able to determine the formula for the volume of a cube which can be found by multiplying the edge times the edge times the edge ($V = r \times r \times r = r^3$). These results were obtained by students after students were able to identify that the length of the base and the width of the base and the height of the shape of the cuboid have the same or similar length so that the three lengths can be said to be an edge (r) which is an element of a cuboid or cube.

The next stage or level that students go through is the formal deduction level. At this level students form a pyramid from a cube and find the formula for the volume of a pyramid from a given cube in the wrong way. What they do is divide the cube into six pyramid parts. For more details about teacher activities and hypotheses from student learning paths are shown in Table 4.

At this level, students can prove or find the formula for the volume of a pyramid derived from the volume of a cube. So that the conclusions obtained by students about the volume of the pyramid can be found by multiplying the area of the base of the pyramid and the height of the pyramid, the result of which is divided by three ($V = 1/3 \times$ Base area $\times$ pyramid height).

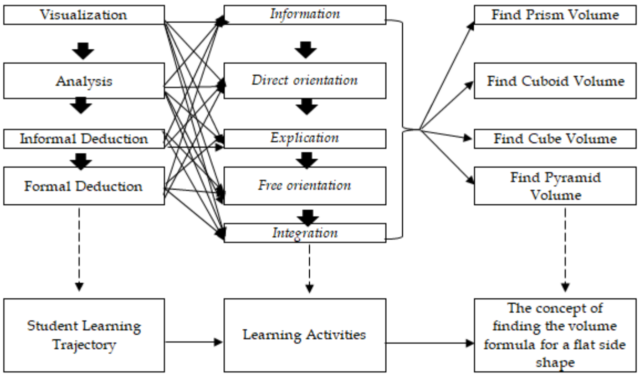

Figure 7. The hypothesis of the flow of thinking of students learning geometry to build flat sided spaces

These four stages are a series of paths that must be passed by students in order to understand the concept of geometrical volumes, namely prisms, blocks, cubes and pyramids. Step by step is very important because it can be used as a reference for further learning. The following scheme illustrates an example of a series of geometric learning activities based on Van Hiele's theory which is hypothesized according to the flow of students' thinking (Figure 7).

Conclusion

Based on the description of the example of the application of the hypothetical learning trajectory (HLT), it can be concluded that the hypothetical learning trajectory volume of a flat side shape can be used as a learning guide for teachers in achieving learning objectives in finding the desired volume formula of cubes, cuboids, prisms and pyramids and becomes one of the alternatives strategies in overcoming difficulties in understanding geometric concepts in learning flat sided shapes in middle schools.

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