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Study Review of the Speed of Light in Space-Time for STEM Student

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Abstract: The author of this article aims to review the theory of relativity and its implications for physics education by using visual aids and a programming approach. The article will cover the concept of the speed of light in space-time in the context of relativity, and provide illustrations that explain the relationships in the context of general relativity. The focus of the article will be to introduce students to complex concepts and encourage their interest in the topic. The author reports success in teaching the basic concept of the speed of light in space-time to both elementary STEM students and high school students. While the theory of relativity has been taught at the secondary school level in some education systems, there is a lack of research on the effectiveness of using visual aids and a programming approach to enhance students' understanding of the concept. This article aims to fill the gap by evaluating the impact of this approach on students' understanding of the theory of relativity.

Keywords: General relativity; Physics education; Speed of light

Introduction

The theory of relativity is a set of physical laws that describe the fundamental nature of space, time, and matter (Wilujeng, 2021). These laws were developed by Albert Einstein in the early 20th century and have had a profound impact on our understanding of the universe (Peebles, 2020). While the theory of relativity is a complex and advanced topic, it is now being studied at a high school level in some educational systems. Didactic insights refer to the understanding and knowledge that a teacher or instructor has about a particular subject or topic (Pandiangan, 2019), They are the insights that allow the teacher to effectively explain and teach the topic to students.

The principle of general covariance is a fundamental principle in the theory of general relativity (GR) (Tessarotto et al., 2021). It states that the laws of physics should be the same for all observers, regardless of their relative motion or the presence of gravitation (Damanik, 2022). In other words, the laws of physics should be *covariant*, meaning that they should remain unchanged when viewed from different reference frames (Giacomini et al., 2019).

The idea that physical laws can be written in a form that does not depend on the coordinate system means that the laws should be the same for all observers (Giacomini et al., 2019), regardless of the particular coordinate system they are using to describe the events in question. In other words, the laws of physics should be *covariant* meaning that they should remain unchanged when viewed from different reference frames (Höhn, 2019). Coordinates are numbers that are used to specify the position of an event in space and time (Schutz, 2022). In physics, we often use a coordinate system to describe the position of an event in four-dimensional spacetime. $\mathbf{R}^4$ consists of all possible combinations of four real numbers (x, y, z, t), which correspond to the spatial coordinates (x, y, z) and the time coordinate (t).

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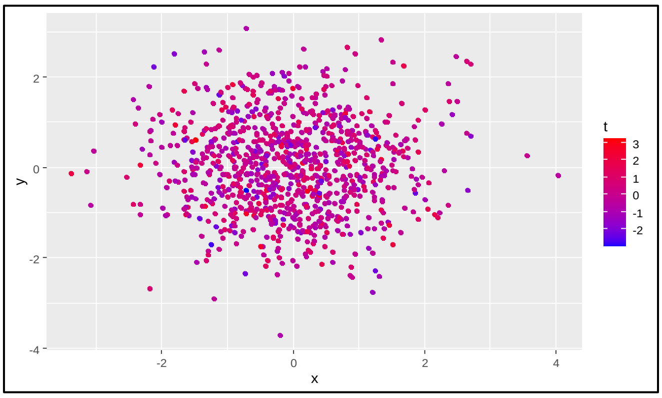

Figure 1. Correspond to the spatial coordinates (x, y, z) and the time coordinate (t) (Ruben Siagian in program r plot, 2023)

The plot above illustrates the concept of relativity using a scatter plot. In his physics, the concept of relativity states that space and time are entities that are interrelated and cannot be separated (Schutz, 2022). In the scatter plot, the displayed points represent a combination of four real numbers representing spatial coordinates (x, y, z) and time coordinates (t). The color of each point shows the value of the time coordinate (t). Thus, this plot illustrates how light travels through space-time and how the perception of time and distance can change depending on the position and speed of the observer. This concept shows that time and distance in space-time can have different values depending on the speed and position of the observer. In this case, light is a good example to understand this concept because it has a constant speed in space-time that is not affected by the speed of the observer.

The speed of a body is a measure of how fast it is moving and is calculated by dividing the distance traveled by the time it takes to travel that distance (Rowcliffe et al., 2016). The formula for calculating speed is: speed = distance traveled/time taken (Susetyo, 2022). In classical physics and special relativity, velocity is defined as the rate of change of an object's position with respect to time (Putri, 2022).

It is a measure of how fast an object is moving and is calculated by dividing the distance traveled by the time it takes to travel that distance (Noonan et al., 2019). In classical physics and special relativity, velocity is a global concept that is defined as the rate of change of an object's position with respect to time (Alstein et al., 2021). It is a measure of how fast an object is moving and is calculated by dividing the distance traveled by the time it takes to travel that distance. In the theory of general relativity (GR), the concept of velocity becomes more complex due to the curvature of spacetime caused by the presence of massive objects (Gair et al., 2013). In GR, the velocity of a body is only defined in an infinitesimally small region of spacetime and is referred to as the *local velocity* (Ter-Kazarian, 2021).

This means that the velocity of a body as measured by an observer located in one region of spacetime may be different from the velocity of the same body as measured by an observer located in a different region of spacetime (Sutanto, 2020). It is not uncommon for the concept of velocity to be introduced in high school physics courses, although the treatment of the topic may vary depending on the specific curriculum. In general, velocity is a measure of the speed and direction of an object's motion (Lambaga, 2019). It is usually defined as the rate of change of an object's position with respect to time. Light always travels at a constant speed of approximately 299,792,458 meters per second in a vacuum, which is the highest speed that any object or signal can travel (Rybczyk, 2015). This constant speed of light is known as the speed of light in a vacuum and is represented by the symbol "c" in physics equations (Shivalingaswamy et al., 2017). The speed of light in a vacuum is a fundamental constant of nature and is not affected by the speed or motion of the source of the light or the observer (Kamphorst et al., 2019).

The speed of free fall is the speed at which an object falls toward the surface of a planet or other celestial body due to the gravitational pull of that body (Anjum et al., 2020). It is a common topic in physics and is often studied in the context of classical mechanics, which is the study of how objects move and interact in the physical world (Harefa, 2021). When analyzing the speed of free fall, it is common to consider two different viewpoints: the perspective of a local observer who is near the object that is falling, and the perspective of an observer who is outside the gravitational field of the planet or celestial body (G. Feng et al., 2020). These two viewpoints can yield different results due to the effects of gravity on the motion of the object (Feynman et al., 2018).

From the perspective of a local observer, the speed of free fall is determined by the acceleration due to gravity, which is the rate at which the object's speed increases as it falls (Jaya, 2020). The acceleration due to gravity is a constant value that is determined by the mass and size of the celestial body (Subhan et al., 2022). On Earth, the acceleration due to gravity is approximately 9.8 meters per second squared (m/s^2) (Sutria et al., 2022), this means that the speed of an object in free fall on Earth increases by 9.8 m/s every second. From the perspective of an observer who is outside the gravitational field of the planet or celestial body, the object appears to be moving in a straight line at a constant speed (Ilyas et al., 2020), this is because the observer is not affected by the gravitational field, so the object's motion appears to be uniform and unchanging. However, from the perspective of a local observer, the object appears to be accelerating as it falls due to the effects of gravity (Miwa et al., 2019). Analyzing the speed of free fall from these two different viewpoints can help students to understand how gravity affects the motion of objects (Zendroto, 2019), and how the laws of physics can appear differently depending on the observer's frame of reference (Kurnia, 2021).

A research gap refers to an area of study where there is a lack of research or an unsatisfactory amount of research that has been conducted on a particular topic. In this article, the author identifies a research gap in the field of physics education regarding the use of visual aids and a programming approach in teaching the theory of relativity to students. The author aims to fill this gap by evaluating the effectiveness of this approach on students' understanding of the theory of relativity. The lack of research on this topic suggests that there may be an opportunity for further investigation and exploration of the potential benefits of using visual aids and a programming approach in teaching this complex scientific concept.

Method

The method of this research is to utilize a variety of trustworthy and proven literary studies to guarantee the accuracy of the data presented in the article. The focus of the article is to provide a comprehensive explanation of the fundamentals of the speed of light in space-time and the theory of relativity. The theory of relativity is often challenging for students to grasp, making this article especially important for STEM students who need to understand the geometry of space-time and its impact on the speed of light, including its characteristics and so on. The article includes numerous illustrations to demonstrate the connections between the theory of relativity and make the complex information more accessible to the reader. The plots used are the author's original work using R programming and were created using data from various credible sources. This article aims to be a useful and insightful resource for readers, especially STEM students and high school students who want to comprehend the basic principles of the theory of relativity and the speed of light in space-time.

Result and Discussion

The principle of energy conservation states that energy cannot be created or destroyed (Liu et al., 2018), only converted from one form to another. In the context of a falling object in a gravitational field, this principle can be used to calculate the rate of fall of the object (Tino et al., 2020). To do this, we can use the equation for the conservation of energy, which states that the total mechanical energy of an object (the sum of its kinetic energy and potential energy) is constant as long as there are no external forces acting on the object (Agarana et al., 2017).

For a falling object in a gravitational field, the potential energy of the object decreases as it falls, while its kinetic energy increases (Yani et al., 2019). The rate at which the object falls (its acceleration) is determined by the balance between these two types of energy. For example, if we have a test body falling in the gravitational field of a massive body, we can use the principle of energy conservation to calculate the rate of fall of the test body. To do this, we would need to know the mass of the test body, the acceleration due to gravity (which is determined by the mass and size of the massive body) (Kaur et al., 2017), and the initial height of the test body above the surface of the massive body (Acharya et al., 2018). Using these values, we can calculate the initial potential energy of the test body and the kinetic energy it gains as it falls. By equating the initial potential energy to the final kinetic energy and solving for the acceleration of the test body (Khalifa et al., 2017), we can determine the rate of fall of the test body in the gravitational field of the massive body (Tino et al., 2020).

$$(v_\tau)^2 = \frac{2GM}{R} \tag{1}$$

The distance "R" in the equation for gravitational potential. The gravitational potential at a point within the gravitational field is directly related to the mass of the massive body and the distance between the point and the center of the massive body (Zotos et al., 2020). If the body in question starts from a standstill and is initially far away from the center of the massive body, it will have a low initial gravitational potential (Inayoshi et al., 2020). As the body falls towards the massive body, it gains kinetic energy and its gravitational potential decreases. The rate at which the body falls (its acceleration) is determined by the balance between its kinetic energy and potential energy, as described by the principle of energy conservation (French, 2017).

If the body starts from a standstill and is initially close to the center of the massive body, it will have a high initial gravitational potential (Hut & Rees, 1992). As the body falls towards the surface of the massive body, it gains kinetic energy and its gravitational potential decreases (Carter et al., 2020). The rate at which the body falls (its acceleration) is again determined by the balance between its kinetic energy and potential energy (Pendrill et al., 2020). Overall, the gravitational potential of a body within the gravitational field of a massive body is inversely proportional to the distance between the body and the center of the massive body (Latif et al., 2022).

This means that the closer the body is to the center of the massive body, the higher its gravitational potential will be, and the further it is from the center, the lower its gravitational potential will be. This relationship between gravitational potential and distance is important for understanding the behavior of objects within the gravitational field of a massive body (Annulli et al., 2020). The speed of the body always increases as it falls, meaning that it is accelerating. The speed of the body reaches its maximum value when the distance from the center of the gravitational field (such as the Earth's surface) has a certain value (Sofue, 2020). The speed of the body is calculated using clocks that are at rest in the gravitational field at the points where the body passes (Zschocke, 2022). These clocks are not moving relative to the gravitational field, so they provide an accurate measure of the speed of the body as it falls (Qin et al., 2021). It is important to note that the speed of the body is not calculated using a clock that is moving with the body. This is because the speed of the body is relative to the gravitational field, and a clock moving with the body would not be at rest relative to the gravitational field.

The time τ measured by clocks at rest in a gravitational field (such as the Earth's surface) is known as proper time. Proper time is the time interval measured by an observer who is at rest in the gravitational field (Singal, 2022). The time t measured by clocks outside the gravitational field, i.e. for an observer at an arbitrarily large distance from M (where M is the center of the gravitational field), is known as coordinate time. Coordinate time is the time interval measured by an observer who is not in the gravitational field.

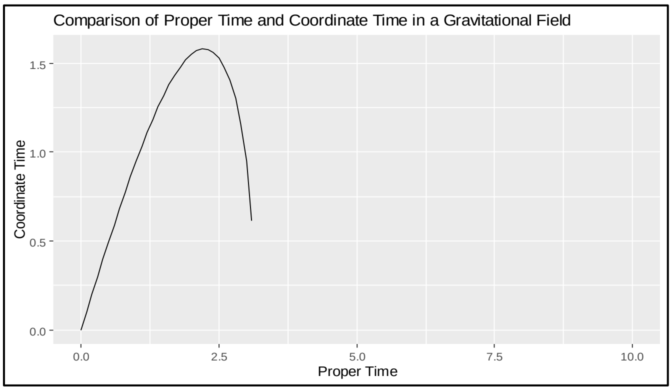

Figure 2. Comparison of proper time and coordinate time in a gravitational field (Ruben Siagian in program r plot, 2023)

There is a link between proper time and coordinate time, and this link is described by the concept of time dilation (Bravo et al., 2023). Time dilation is a phenomenon that occurs when two clocks are moving relative to each other, or when one clock is in a stronger gravitational field than the other (Paczos et al., 2022). Time dilation predicts that the time interval measured by a clock in a stronger gravitational field or moving at a higher velocity will be shorter than the time interval measured by a clock in a weaker gravitational field or moving at a lower velocity (Roura et al., 2021). In the case of a clock in a gravitational field, the proper time measured by the clock will be shorter than the coordinate time measured by an observer at an arbitrarily large distance from the center of the gravitational field (Tanaka et al., 2021).

This means that the clock in the gravitational field will appear to be running slower than the clock outside the gravitational field. The link between proper time and coordinate time is described mathematically by the Schwarzschild metric, which is a mathematical model used to describe the gravitational field around a massive object such as a planet or a star.

$$\tau^2 = t^2\left[1 - 2\frac{GM}{c^2R}\right] \tag{2}$$

The *apparent velocity* refers to the velocity of an object as it appears to an observer. It is calculated by taking the rate of change of the object's distance (R) with respect to time (t). The apparent velocity of an object can be different from its actual velocity, which is the velocity of the object relative to its own frame of reference (Qiu et al., 2018). The apparent velocity of an object is affected by various factors, including the motion of the observer, the gravitational field in which the object is moving, and the relative velocity of the object and the observer.

$$\frac{dR}{dt} = v_t = \tau^2 = t^2\left[1 - 2GMc^{-2}R^{-1}\right] = \frac{d\tau}{dt}\frac{dR}{d\tau} \tag{3}$$

And we have:

$$vt^2 = \frac{2GM}{R}\left[1 - \frac{2GM}{c^2R}\right] \tag{4}$$

The Schwarzschild radial coordinate (R) is a mathematical quantity used to describe the position of an object in a gravitational field (Fiziev, 2019). It is named after Karl Schwarzschild, who first derived the mathematical solution that describes the gravitational field around a massive object, such as a planet or a star. The Schwarzschild radial coordinate is typically used to describe the position of an object in a spherical coordinate system, where the object is located at a distance R from the center of the coordinate system (Ruchlin et al., 2018). The Schwarzschild radial coordinate is often used in conjunction with the polar angle (θ) and the azimuthal angle (φ) to fully specify the position of an object in three-dimensional space.

In the context of explaining the concept of apparent velocity to students, it is common to consider the Schwarzschild radial coordinate as simply a measure of spatial distance, without discussing its more technical definition. This is done for simplicity and to focus on the main concept being taught, which is the relationship between the rate of change of distance and time as measured by an observer.

However, it is important to note that the Schwarzschild radial coordinate is not simply a measure of spatial distance in the same way that distance is typically defined (Tino et al., 2020). It is a mathematical quantity that takes into account the curvature of spacetime caused by the presence of a gravitational field. As such, the Schwarzschild radial coordinate has a more complex definition and is used in more advanced calculations involving the effects of gravity on the motion of objects (Bacchini et al., 2018).

$$\frac{dR}{dv_t} = \frac{\sqrt{\left[\frac{2GM}{R}\left\{1-\frac{2GM}{c^2R}\right\}\right]}(cR)^4\,2}{2GMcR\left(-1C^3R+4GMc\right)} \quad (5)$$

The real velocity (v_τ) of an object is the velocity of the object relative to its own frame of reference. It is measured using clocks that are at rest in the gravitational field at the points where the object passes. The apparent velocity (v_t) of an object is the velocity of the object as it appears to an observer. It is calculated by taking the rate of change of the object's distance (R) with respect to time (t) as measured by the observer. In the case described, the real velocity ($v\tau$) of the object starts at zero (at an infinite distance from the center of the gravitational field) and increases to a maximum value of c (the speed of light) at a certain distance $R = R_s$ from the center of the gravitational field.

This behavior is expected, as the object is accelerating due to the force of gravity and its velocity increases as it falls. The apparent velocity (v_t) of the object has a different behavior (Hinds et al., 2022). It starts at zero (at an infinite distance from the center of the gravitational field), increases to half the speed of light, and then decreases back to zero. This behavior is due to the effect of time dilation, which causes the time interval measured by the observer to appear shorter than the time interval measured by the object (Pons et al., 2018).

As a result, the apparent velocity of the object appears to decrease as it falls, even though its real velocity is increasing. It is important to note that the real velocity and the apparent velocity of an object are not the same, and they can behave differently due to the effects of time dilation and other factors such as the relative motion of the object and the observer.

The apparent velocity (v_c) of a photon refers to the velocity of a photon as it appears to an observer. It is calculated by taking the rate of change of the photon's distance (R) with respect to time (t) as measured by the observer. The velocity of a photon is a unique case, as it always travels at the speed of light (c) in a vacuum. This means that the real velocity ($v\tau$) of a photon is always equal to c, regardless of the frame of reference or the presence of a gravitational field.

However, the apparent velocity (v_c) of a photon can be different from its real velocity due to the effect of time dilation. Time dilation is a phenomenon that occurs when two clocks are moving relative to each other, or when one clock is in a stronger gravitational field than the other (Paczos et al., 2022). Time dilation predicts that the time interval measured by a clock in a stronger gravitational field or moving at a higher velocity will be shorter than the time interval measured by a clock in a weaker gravitational field or moving at a lower velocity.

In the case of a photon, the apparent velocity (v_c) of the photon will be affected by the relative motion of the photon and the observer, as well as the gravitational field in which the photon is moving. If the photon is moving through a stronger gravitational field or is moving at a higher velocity relative to the observer, the apparent velocity (v_c) of the photon will be less than c (Šorli et al., 2021). It is important to note that the apparent velocity (v_c) of a photon is not the same as the real velocity (v_τ) of the photon, and it is affected by the relative motion and gravitational field of the photon and the observer.

$$\frac{d\tau}{dt}\frac{dR}{d\tau} = v_c \quad (6)$$

Therefore,

$$\frac{dR}{d\tau} = \sqrt{c^2 - \frac{2GM}{R}} \quad (7)$$

This statement refers to the behavior of a photon as it approaches the event horizon of a black hole. The event horizon is a boundary around a black hole beyond which it is not possible to escape, due to the extreme gravitational forces that exist within the event horizon (Rummel et al., 2020). For an external observer, the apparent velocity (v_c) of a photon will appear to decrease as the photon approaches the event horizon of a black hole. This is due to the effect of time dilation, which causes the time interval measured by the observer to appear shorter than the time interval measured by the photon. As the photon moves closer to the event horizon, it experiences a stronger gravitational field, which causes the time interval measured by the photon to appear longer to the observer. As a result, the apparent velocity (v_c) of the photon appears to decrease as it approaches the event horizon (Chael et al., 2021).

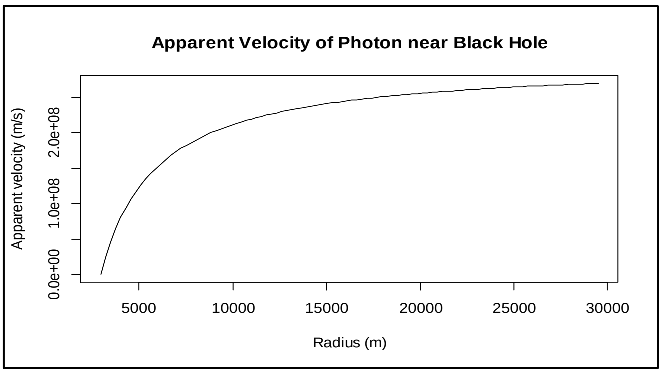

Figure 3. Apparent velocity of photon near blackhole *(Ruben Siagian in program r plot, 2023)*

It is important to note that the apparent velocity (v_c) of a photon is not the same as the real velocity (v_τ) of the photon. The real velocity of the photon is always equal to c, the speed of light, regardless of the presence of a gravitational field (HenokTadesse et al., 2018). However, the apparent velocity of the photon is affected by the relative motion and gravitational field of the photon and the observer, and it can appear to be different from the real velocity as measured by the observer. In the case described, the apparent velocity of the photon appears to slow down as it approaches the event horizon of a black hole (Dokuchaev, 2019). However, the real velocity of the photon remains constant at c, the speed of light.

$$\begin{cases} \lim_{R\to\infty} v_c = c \\ \lim_{R\to R_s} v_c = 0 \end{cases} \tag{8}$$

Gravitational time dilation is a phenomenon that occurs when an object is in a strong gravitational field. It is based on the idea that time passes at different rates in different gravitational fields, with time passing more slowly in stronger gravitational fields (Xia et al., 2019). In the case of circular orbits, an object (such as a satellite) is moving in a circular path around a much more massive object (such as a planet) (Rachman, 2021). The gravitational field of the more massive object causes the satellite to experience gravitational time dilation (Astro et al., 2019).

This means that the satellite's clock will appear to run slower to an observer on the surface of the more massive object, compared to the clock of an observer who is farther away from the gravitational field. This effect is due to the fact that the satellite is constantly accelerating as it moves in its circular orbit, and this acceleration results in a difference in the passage of time (Colmenero et al., 2021). The students may have proposed to analyze this case because they are interested in understanding how gravitational time dilation works in more detail, and how it might affect the behavior of objects in circular orbits.

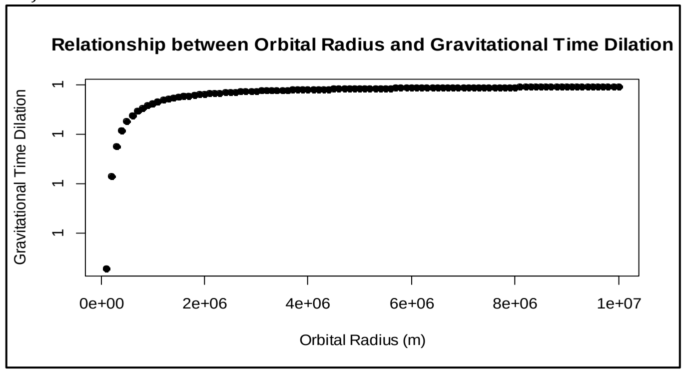

Figure 4. Relationship between orbital radius and gravitational time dilatation (Ruben Siagian in program r plot, 2023)

$$\begin{aligned} \tau^2 &= t^2\left[1-3GMc^{-2}R^{-1}\right] \\ &= t^2\left[1-\frac{3}{2}R_sR^{-1}\right] \end{aligned} \tag{9}$$

The photon sphere is a region around a massive object (such as a black hole) where the gravitational force is so strong that it can cause photons (particles of light) to be trapped in a circular orbit.

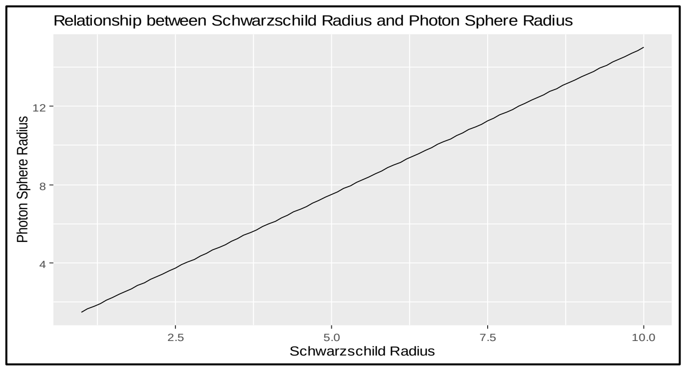

Figure 5. Relationship between schwarzschild radius and photon sphere radius (Ruben Siagian in program r plot, 2023)

The radius of the photon sphere is defined as 3/2 times the Schwarzschild radius of the object, where the Schwarzschild radius is a measure of the size of the object's event horizon (the boundary beyond which nothing, not even light, can escape).

In physics, the photon sphere is a region around a massive object (such as a black hole) where the gravitational force is so strong that it can cause photons (particles of light) to be trapped in a circular orbit. The radius of the photon sphere is defined as 3/2 times the Schwarzschild radius of the object, where the Schwarzschild radius is a measure of the size of the object's event horizon (the boundary beyond which nothing, not even light, can escape). The Newtonian circular velocity is the velocity that an object would need to have in order to maintain a circular orbit around a massive object, based on Newton's laws of motion. The equation for calculating the Newtonian circular velocity takes into account the gravitational constant (G) and the mass of the object being orbited (mass) as well as the radius of the orbit (radius). The simulation in calculates the relationship between the mass of a black hole and the Newtonian circular velocity of an object around it by using the functions schwarzschild radius, photon sphere radius, and newtonian circular velocity.

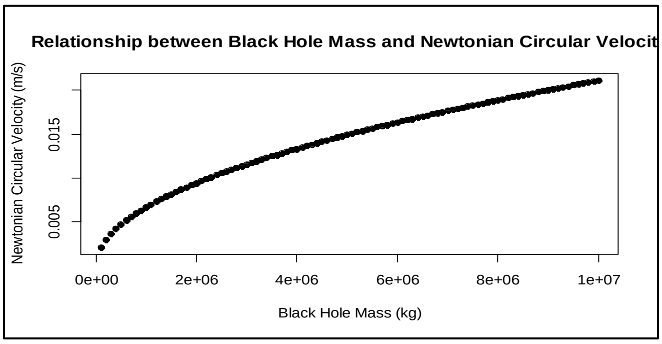

Figure 6. Relationship between blackhole mass and newtonian circular velocity (Ruben Siagian in program r plot, 2023)

In physics, the photon sphere is a region around a massive object (such as a black hole) where the gravitational force is so strong that it can cause photons (particles of light) to be trapped in a circular orbit. The radius of the photon sphere is defined as 3/2 times the Schwarzschild radius of the object, where the Schwarzschild radius is a measure of the size of the object's event horizon (the boundary beyond which nothing, not even light, can escape). The Newtonian circular velocity is the velocity that an object would need to have in order to maintain a circular orbit around a massive object, based on Newton's laws of motion. The equation for calculating the Newtonian circular velocity takes into account the gravitational constant (G) and the mass of the object being orbited (mass) as well as the radius of the orbit (radius). The simulation in calculates the relationship between the mass of a black hole and the Newtonian circular velocity of an object around it by using the functions schwarzschild radius, photon sphere radius, and newtonian circular velocity.

The Newtonian circular velocity is the velocity that an object would need to have in order to maintain a circular orbit around a massive object, based on Newton's laws of motion (J. Q. Feng, 2020). It is given by the formula $v = [\frac{GM}{r}]^{\frac{1}{2}}$, where G is the gravitational constant, M is the mass of the object being orbited, and r is the distance from the object's center of mass. The correction to the Newtonian circular velocity refers to a modification of this formula that takes into account the effects of general relativity, which is a theory that describes how gravity works on a larger scale (Ruggiero et al., 2022). General relativity predicts that the velocity needed to maintain a circular orbit around a massive object will be slightly different than the velocity predicted by Newton's laws, due to the curvature of spacetime caused by the object's gravitational field.

The correction to the Newtonian circular velocity is typically expressed in terms of a parameter called the "*gravitational redshift factor*" which takes into account the difference between the passage of time in the gravitational field of the object and the passage of time in a region where the gravitational field is weaker. The gravitational redshift factor can be used to calculate the actual velocity needed to maintain a circular orbit around the object, taking into account the effects of general relativity.

$$\frac{(R-R_s)}{GM} v_\tau^2 = 1 \quad (10)$$

In this context, v is the velocity of the photon, c is the speed of light, and τ is a measure of the time it takes for the photon to complete one orbit around the mass. If we set v_τ equal to c, this means that the time it takes for the photon to complete one orbit is equal to the time it would take light to travel a distance equal to the circumference of the orbit (Brown, 2018). The Schwarzschild radius (R_s) is a measure of the size of the event horizon of a black hole. It is defined as the distance from the center of the black hole at which the escape velocity of an object becomes equal to the speed of light. When the distance R from the center of the mass to the photon is equal to $3/2R_s$, this means that the photon is orbiting the mass at a distance of 3/2 times the Schwarzschild radius.

This distance is often referred to as the "photon sphere" because it is the distance at which photons can orbit around the mass. The photon sphere is an important feature of black holes because it is the point at which the gravitational field becomes so strong that photons can no longer escape (Gan et al., 2021). This means that anything that falls within the photon sphere will be unable to escape the black hole.

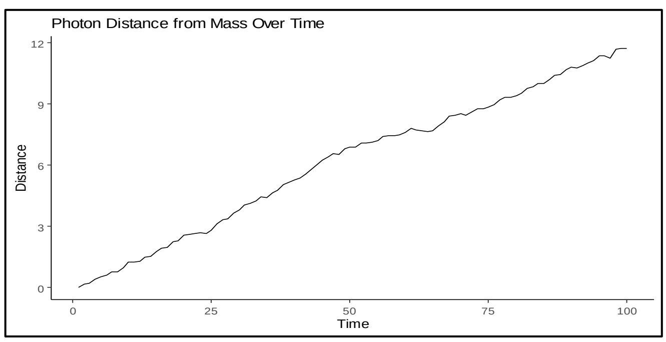

Figure 7. Photon distance from mass over time (Ruben Siagian in program r plot, 2023)

V_c^2 will get a value of 0, because $\frac{d\tau}{dt}\frac{d(R.\theta)}{d\tau}$ equal with $\frac{d(R.\theta)}{d\tau}$. The concept of distance and velocity in an expanding universe can be challenging to understand, especially at the bachelor STEM level. In an expanding universe, the distance between objects increases over time, and this can make it difficult to understand how velocities are defined and measured.

In the context of a photon appearing to not revolve around a mass as if it were frozen, this may be due to the fact that the expansion of the universe is causing the distance between the photon and the mass to increase over time (Kolb et al., 2021). This means that the velocity of the photon may not appear to change, even though it is actually moving in a circular orbit around the mass.

One way to think about this is to consider the expansion of the universe as a kind of *Stretching* of space (Kurki, 2020). As the space between the photon and the mass stretches, it may appear to an observer that the photon is not moving, even though it is actually orbiting the mass (Becerril et al., 2021). It is worth noting that the concept of distance and velocity in an expanding universe is a topic of ongoing research in cosmology, and our understanding of these concepts is constantly evolving (Damanik, 2022; Dodelson et al., 2020).

Conclusion

In this paper, the authors describe a lecture they gave to high school students on the concept of velocity in general relativity. They started by discussing how an observer at a large distance from a massive body in free fall in a gravitational field would measure a different rate of fall from the actual velocity of free fall. This allowed them to introduce the concept of a black hole, which is a region of space where the gravitational field is so strong that nothing, not even light, can escape. The authors then focused on the speed of light and discussed both radial and circular motion. They used mathematical formalism appropriate for the students' age to try to understand how the apparent speed of light can be affected by time dilation, a phenomenon that occurs when time appears to move at a different rate in different reference frames. In some cases, the light can even appear to be frozen due to the time dilation. Overall, the goal of the lecture was to introduce the students to some of the complex concepts in general relativity and stimulate their interest in the topic. It seems that the lecture was successful in achieving this goal, as the students were reportedly very fascinated by Einstein's theory.

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