JPPIPA 7(2) (2021)

Jurnal Penelitian Pendidikan IPA

*Journal of Research in Science Education*

http://jppipa.unram.ac.id/index.php/jppipa/index

# Physics Model for Rope Jumping Game

Fathan Akbar[1], Ardi Khalifah[2], Mikrajuddin Abdullah[3*]

[1] Program Studi Fisika, Institut Teknologi Bandung, Jl. Ganesa 10 Bandung 40132, Indonesia.
[2] SMA PGII 1, Bandung, Jl. Panatayuda 2 Bandung 40132, Indonesia.
[3] Pusat Fisika Interdisipliner, Program Studi Fisika, Institut Teknologi Bandung, Jl. Ganesa 10 Bandung 40132, Indonesia.

DOI: 10.29303/jppipa.v7i2.559

**Article Info**
Received: January 5th, 2021
Revised: April 18th, 2021
Accepted: April 22th, 2021

**Abstract.** We developed a simple physics model to explain the profile of the rope played in the rope jumping game. Firstly, we derived a second order non-linear differential equation to explain the rope motion. Analytical solutions can be obtained if the displacement of all points along the rope is small. For arbitrary deviations, a numerical solution must be employed, and at the present paper, we used a simple excel-visual basic program. We found that the profiles of the simulation results is very similar to the real profile which can be observed in a number of sources on the internet. A critical value separating the condition where the rope length remains unchange and the condition when the rope changes suddenly with $\rho\omega^2/T$ was identified. A scaling relationship was also identified in the changing region with the critical exponent of -0.31. The existence of the critical point and the critical exponent in the changing region informs that the change in the rope profile resembles the phase transition phenomenon.

**Keywords**: Rope jumping; visual basic; critical point; scale factor; phase transition

**Citation:** Akbar, F., Khalifah, A., & Abdullah, M. (2021). Physics Model for Rope Jumping Game. *Jurnal Penelitian Pendidikan IPA, 7*(2), 281-286. doi:https://doi.org/10.29303/jppipa.v7i2.559

## Introduction

There are a number of ancient children's games that are starting to be forgotten today. The very rapid development of IT technology causes children today to spend more time with games available on devices. Majority of children living in this era are unfamiliar with playing marbles, cranks, jumping rope, bamboo rifles, hornbills, congklak, and so on. They are more familiar with mobile legends, point blank, minecraft, super mario, and the like. “Play together” are words that are very familiar to children today that they play games together online. Participants "playing together" can come from various cities, even various countries.

One of the old children's games that appealed to us was jumping rope. The rope can be rotated by two people, each twisting the ends, and the third person jumps in the middle (Azka, 2019; Jeenall, 2018; Channel, 2019). Another form is skipping where one person spins the rope by holding two ends of the rope while jumping (Homerie, 2018; Susanto, 2018; Tube, 2019). This game is interesting to study because there are physics equations that can be applied to explain the shape of the rope when it is rotated.

Indeed, there has been a very brief discussion of the skipping phenomenon, but it did not arrive at the simulation results. Furthermore, the resulting equation contains the elliptic function which is quite complicated (Ferréol, 2017). A deeper discussion by examining the dynamics and shape of the rope during skipping has been reported by Aristoff and Stone (2012). At the present work, however, we reformulate the equation so that it can be understood by both teachers and students. We will also perform numerical calculations using excel-visual basic to predict the shape of the string when rotated as a function of a number of parameters.

Physical study of the phenomenon around them, such as past children's games or practices carried out by

Email: mikrajuddin@gmail.com

ancient people, are sometimes very challenging. From this study, it is possible to emerge new understandings or new theories. We have reported many studies related to these matters (Abdullah, et. al., 2014; Rahmayanti, et. al., 2016; Yuliza, et. al., 2018; Sinebar & Abdullah, 2018; Margaretta, et al., 2019; Akbar & Abdullah, 2020).

## Modeling

Consider Figure 1. If the rope forms a formation at the bottom (at the minimum position) then the resultant force acting on the rope element $dm$ is

$$dF_b = T\,sin\theta(x) - Tsin\theta(x+d) - dm\,g \quad (1)$$

where $T$ is the tension of the rope, $\theta$ is the slope at position $x$, and $g$ is the acceleration of gravity. The force takes an upward direction. Conversely, if the rope takes the formation above (is at the maximum position) then the net force acting on the element is $dF_u = T\,sin\theta(x) - Tsin\theta(x+d) + dm\,g$. We note that the force originates from the tension in the rope at the two ends of the element and the gravitational force directs downward. The tension force of the rope on the element depends on the angle of inclination of the rope at each end of the element.

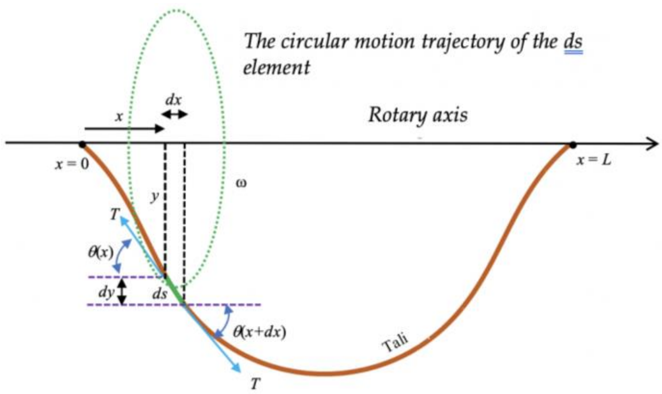



**Figure 1.** Illustration when the rope is in formation below.

Let's focus on the conditions when the rope is in formation at the bottom. The resulted equation can be directly applied to the condition of the rope forming the upper formation simply by replacing the gravitational acceleration with its negative. If the rope is rotated at angular velocity $\omega$ about the horizontal axis, the mass element $dm$ satisfies the following equation

$$dF_b = dm\omega^2 y \quad (2)$$

where $y$ is the perpendicular distance of the mass element to the axis of rotation (acting as the radius of motion for the element dm). We assume that the rope has a homogeneous mass distribution (density per unit length, $\rho$, constant). If $ds$ is the length of the rope element then $dm = \rho\,ds = \rho\sqrt{dx^2 + dy^2} = \rho\sqrt{1+y'^2}dx$. The angle of inclination of the rope at position $x$, which is $\theta(x)$, satisfies the following equation

$$\sin\theta(x) = \frac{dy}{ds} = \frac{dy}{dx\sqrt{1+y'^2}} = \frac{y'}{\sqrt{1+y'^2}} \quad (3)$$

An additional assumption that we apply is the rope tension is constant along the rope. The contribution of the rope tension to the $dm$ element is solely the result of the difference in the slope of the tangent to the two ends of the element. By using equations (1) - (3), we can derive the following equation,

$$y" + \frac{\rho\omega^2}{T}\left(1+y'^2\right)^{3/2}(y+\tilde{g}) = 0 \quad (4)$$

where we define $\tilde{g} = g/\omega^2$.

For convenience, let's define temporary variables $z = y + \tilde{g}$. Based on this definition it is easy to prove that $z' = y'$ and $z" = y"$. Thus, equation (4) can be rewritten as

$$z" + \frac{\rho\omega^2}{T}\left(1+z'^2\right)^{3/2} z = 0 \quad (5)$$

Equation (5) is a non-linear differential equation. The solution to this equation cannot be obtained analytically. The solution to the equation must be solved numerically. However, before we look for the general solution to equation (5), we will discuss two special cases where the deviation of all the points along the rope is very small and the specific solution around the knot point of the rope.

**The special case of rope drift is very small**

The case of very small rope deviation is often applied when we discuss the propagation of rope waves (Abdullah, 2017). With this approximation we can simplifysin $\theta(x) \approx \theta(x) \approx dy/dx$. However, because the deviation is very small, $|dy/dx| \ll 1$there are implications$z' \ll 1$. Thus $z'$ it can be neglected compared to 1 and equation (5) can be simplified to

$$z" + \frac{\rho\omega^2}{T} z \approx 0 \quad (6)$$

Equation (6) is a simple harmonic equation of motion and the solution is very well understood, namely, $z = A\sin(\kappa x + \varphi_0)$ where, $\kappa = \omega\sqrt{\rho/T}$, $A$ and $\varphi_0$ are constants of integration.

Next, we apply the boundary conditions. For example, the length of the horizontal direction from end to end of the rope is $L$. One end is at position $x = 0$x and the other end is at position $x = L$. Because the two ends of the rope are at the same height $z(0) = z(L)$. By using the general solution obtained and considering that the two ends have a slope with a different sign (the slope at $x = 0$ is negative and at $x = L$ is positive) this condition is fulfilled if only a half wave and is formed $\kappa L = \pi$. With this condition, the tension in the rope fulfills the equation

$$T = \frac{\rho\omega^2 L^2}{\pi^2} \quad (7)$$

From equation (7) it is clear that the tension in the rope is directly proportional to the square of the length and frequency of rotation.

There have been several instances of extra high voltage power lines breaking due to strong winds. In the news of Kompas (2008), when heavy rain and strong winds hit Bekasi City, the extra high-tension cable which is right above the Kranji flyover, Kalibaru Village, Medan Satria District, was cut off. Tribunnews News (2012), an extra-high voltage cable broke and hit the roof of a resident's house in Leuweunggede Village, RT 3 RW 10, Situwangi Village, Cihampelas District, West Bandung Regency. Previously, strong winds hit the area from noon to evening. Strong winds are thought to produce a significant vibration frequency that increases the cable tension. If the cable tension exceeds the maximum strength, the cable may break. Furthermore, the length of the extra high-tension cable (the distance between the poles is quite far) thus increases the cable tension quadratically to the length.

**Special case solution near the end of the strap**

Because the deviation of the ends of the rope is zero, the deviation around the ends of the rope is very small. At these positions which $y \ll \tilde{g}$ so that equation (4) becomes

$$y'' + \frac{\rho\omega^2\tilde{g}}{T}\left(1 + y'^2\right)^{3/2} \approx 0 \quad (8)$$

We look for a general solution to this equation using Wolframalpha (2020) and we get

$$y' = \pm i \frac{(ax - c_1)}{\sqrt{(ax - c_1)^2 - 1}} \quad (9)$$

where $a = \rho\omega^2\tilde{g}/T = \rho g/T$ and $c_1$ are integration constants. The constant $c_1$ determines the slope of the rope at x = 0. The slope at the position satisfies $y'(0) = \pm i c_1/\sqrt{c_1^2 - 1}$. Since the slope must be a real number whereas in this expression there is an imaginary factor, the imaginary factor is lost only if $c_1^2 - 1 < 0$ or $|c_1| < 1$ .This requirement results in $y'(0) = \pm c_1/\sqrt{1 - c_1^2}$ or $c_1 = y'(0)/\sqrt{1 + y'(0)^2}$. It is very clear from the last relationship that $|c_1| < 1$, which is the expected condition.

Now that we have the solution for the gradient, we next look for a solution for the drift. We will again use Wolframalpha (2020) in equation (9) and get

$$y(x) = c_2 \pm \frac{1}{a}\sqrt{1 - (ax - c_1)^2} \quad (10)$$

where $c_2$ is the second integration constant. The boundary condition is $y(0) = 0$so that $c_2 = \pm(1/a)\sqrt{1 - c_1^2}$. From this we conclude that both $c_1$ and $c_2$ are determined by the slope of the ends of the rope.

Equation (10) can only be applied to 0 $x \le x_m$ where $x_m$ satisfies equation $1 - (ax_m - c_1)^2 = 0$ to make $y(x)$ real. This similarity results

$$x_m = \frac{1 + c_1}{a} = (1 + c_1)\frac{T}{\rho g} \quad (11)$$

**General Solutions**

There is no analytical method that we can apply to find the general solution to equation (5). The only way is by numerical method. However, we will not use difficult numerical methods and do not use special numerical software to find the solution. We will only use excel combined with macros. Macros in Excel are prepared using a visual basic program. Although there are a lot of visual basic instructions, for our case now, we just need to make use of simple instructions. The first instruction is a calculation in a loop and displaying the results in an excel spread sheet. The calculation results are drawn in excel.

We choose Excel because almost all computers used in various countries contain MS Office. The ecxel is one of the applications in MS Office. Thus, lecturers, teachers, students, or even students can run similar computing programs without the need to install other applications.

To solve equation (5) numerically, we must first convert the equation to discrete form. We will use a transformation that is well known (Abdullah, 2020), namely

$$z'' \to \frac{z_{i+2} - 2z_{i+1} + z_i}{\Delta x^2} \quad (12a)$$

$$z' \to \frac{z_{i+1} - z_i}{\Delta x} \quad (12b)$$

$$z \to z_{i+1} \quad (12c)$$

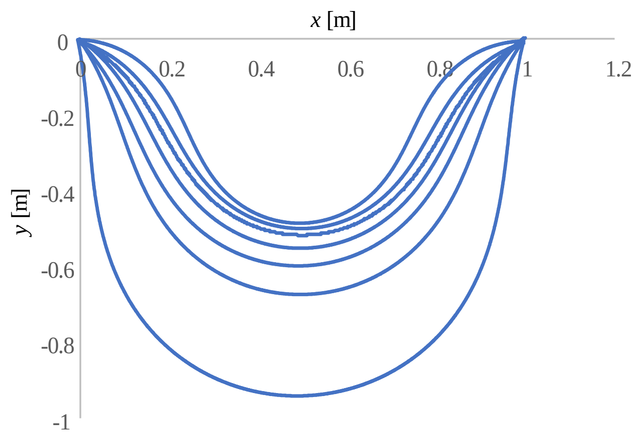
We substitute equation (12a) - (12c) into equation (5) to obtain the following recursive equation

$$z_{i+2} = 2z_{i+1} - z_i - \frac{\rho\omega^2}{T}\left(1 + \left(\frac{z_{i+1}-z_i}{\Delta x}\right)^2\right)^{3/2} z_i \Delta x^2 \quad (13)$$

The boundary conditions used are $z_0 = y(0) + \tilde{g} = \tilde{g}$ and $(z_1 - z_0)/\Delta x = y'(0)$ or$z_1 = \tilde{g} + y'(0)\Delta x$. In our calculations, we have to do trial and error to determine T so that $z_N \approx z_0$ is met.

## Simulation and Discussion Results

Figure 2 shows the rope profile in the formation below (left) and the formation above (right). In the simulation, we maintain the horizontal distance of the two ends of the rope at 1 meter and the angular velocity of rotation is $2\pi$ rad/s. We see that the resulting rope profile is very similar to the real profile (Shinta, 2018; Azka, 2019; Jeenall, 2018; Channel, 2019; Homerie, 2018; Susanto, 2018; Tube, 2019). In both formations (above and below), the curve of the rope and the length of the rope increase as the absolute value of the angle at the end of the rope increases. We also get that the rope profiles when the angle is changed from 0° to 15° are almost coincide (not shown in the figure). Changes only begin to occur when the tip angle reaches about 20° and changes rapidly when the angle is above 20°. We suspect that the 20° angle is like the critical angle where the length of the rope barely changes below that value and changes rapidly above that value. We also observe from Figure 2 that the upper profile is a reflection (reflection) of the bottom profile about the x-axis.



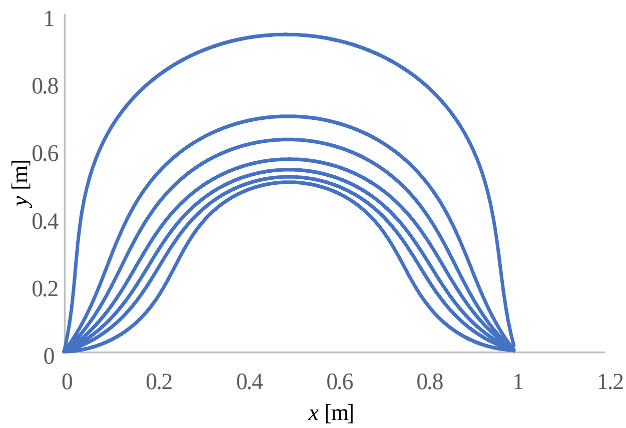




**Figure 2.** The profile of the rope shape when the formation is below (left) and the formation above (right). The distance between the two ends is maintained at 1 meter and the rotating speed is $2\pi$ rad / s. (left) The tilt angle of the ends of the rope from top to bottom is -2,5°, -20°, -30°, -40°, -50°, -60° and-80°. (right) The angle of inclination of the ends of the rope from bottom to top is 2,5°, 20°, 30°, 40°, 50°, 60°, and 80°

Figure 3 is the dependence of the rope length on the absolute value of the angle of inclination at the end of the rope. Circles are simulated results for ropes in the bottom formation and stars are simulated for ropes in the upper formation. The distance between the two ends of the rope is maintained at 1 meter and the angular velocity of rotation is $2\pi$ rad / s. We observe that the length of the rope is the same for the same angle of inclination (absolute value). From this result we will also conclude that wherever the rope formation (up, down, front, back, or any other position), the form of the junction is exactly the same. When the rope is rotated, the surface formed by the rope is in the form of a rotating object surface. There is no deviation of the rope profile when it is rotated.

Figure 4 shows the dependence of the rope length on $\rho\omega^2/T$ for the rope forming the formation below. The two quantities are expressed on a natural logarithmic scale. For the rope forming the above formation, the corresponding values for $L$ and $\rho\omega^2/T$ are almost the same so that we will get a similar curve.

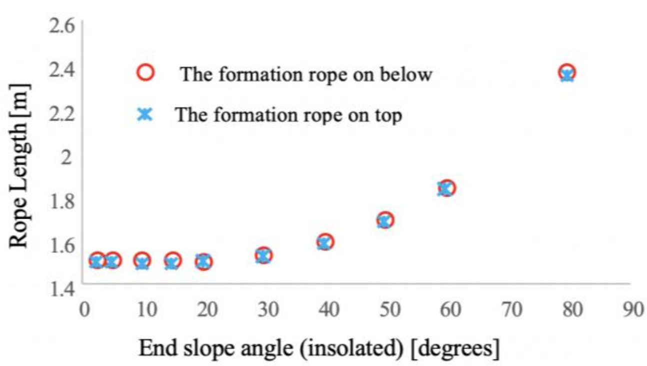



**Figure 3.** The dependence of the length of the rope to the absolute value of the angle of inclination at the end of the rope. Circles are simulated results for ropes in the formation below and stars are simulated for ropes in the formation above.

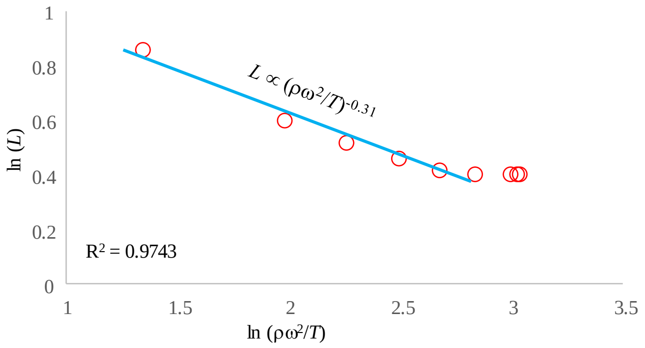



**Figure 4**. Hanging rope length against $\rho\omega^2/T$ for the rope forming the formation below. The two quantities are expressed on a natural logarithmic scale. For the rope forming the above formation, the corresponding values for L and $\rho\omega^2/T$ are almost the same so we will get a similar curve. Linear curves are fitting results involving only the six data sets on the left.

We can see from Figure 4 that as $\ln(\rho\omega^2/T)$ increases, $\ln(L)$ initially decreases linearly until $\ln(\rho\omega^2/T) \approx 2.83$ then nearly unchanges with the increase in $\ln(\rho\omega^2/T)$. We can expect that this value is a critical value for $\ln(\rho\omega^2/T)$ where above this values the length of the rope hardly changes and only below that value the length of the rope changes. The critical value is equivalent to

$$\frac{\rho\omega^2}{T} \approx 17 \quad (14)$$

We get from equation (14) the relationship between three quantities, namely density per unit length, rotational angular velocity, and rope tension. If one of the quantities is changed, the other two quantities are also changed to obtain a critical value.

If we do a string length fitting below the critical value then we get a straight line as in Figure 4. The fitting line has a slope of -0.31 with R2 = 0.9743. Thus, the equation relating the length of the rope and $\rho\omega^2/T$ below the critical value is $\ln L = b_1 - 0{,}31 \ln(\rho\omega^2/T)$ or

$$L \propto \left(\frac{\rho\omega^2}{T}\right)^{-0{,}31} \quad (15)$$

From the above discussions, we simplify that the game of jumping rope that was often played by children in the past has an interesting physical content to study. We get some sort of critical value and get a scale equation with the critical exponent of -0.31. The existence of a critical value and a scale factor is a characteristic of the phase transition phenomenon. Thus, in jumping there is a phase transition phenomenon in which the length of the rope, which was constant, suddenly changes drastically when it exceeds a certain critical value.

## Conclusion

We have proven that the profile of a rope rotated in a skipping game gives birth to a non-liner differential equation. The solution for the case of small deviation gives the equation for sinusoidal deviation. The solution for large deviations must be treated numerically. We show that the deviation profiles in the various rope formations (top, bottom, front, rear and others) are almost the same. We also identified the existence of scale equations and critical exponents similar to those found in phase transition phenomena. Therefore, the rope that is rotated in this game represents a kind of phase transition phenomenon.

## References

Abdullah, M. (2017). *Fisika Dasar II*. tidak dipublikasikan. [Indonesian]

Abdullah, M. (2020). Diambil dari profmikra.org. [Indonesian]

Abdullah, M., Khairunnisa, S., & Akbar, F. (2014). Bending of sparklers. *European Journal of Physics, 35*, 035019. Doi: https://doi.org/10.1088/0143-0807/35/3/035019

Akbar, F. & Abdullah, M. (2020). A sarong rolled around a body demonstrates the force for separating two sheets joined by folding and rolling is very large. *Physics Education, 55*, 065020. Doi: https://doi.org/10.1088/1361-6552/abae24

Aristoff, J.M. & H. A. Stone, H.A. (2012). The aerodynamics of jumping rope. *Proceedings of the Royal Society A, 468*, 720–730. doi: https://doi.org/10.1098/rspa.2011.0389

Azka, S. (2019). *Seru mainan anak tradisional lompat tali - tarik tambang dan engklek - jump rope for kids*. Retrieved from: https://www.youtube.com/watch?v=HCNzuzSdPFo&t=171s. [Indonesian]

Channel, MG. (2019). *Masih ingat ?? 9 Permainan jaman dulu yang terlupaka*. Retrieved from: https://www.youtube.com/watch?v=I8_yMhwELKw. [Indonesian]

Ferréol, R. (2017). *Skipping Rope Curve*. Retrieved from: https://mathcurve.com/courbes2d.gb/cordeasauter/cordeasauter.shtml.

Homerie (2018). *Yuk, belajar lompat tali*. Retrieved from: https://www.youtube.com/watch?v=WliPjgOlnF0. [Indonesian]

Jeenall, M. (2018). *Lompat tali - permainan tradisional.* Retrieved from: https://www.youtube.com/watch?v=eUF3woqI7oQ&t=165s. [Indonesian]

Kompas.com (2008). *Kabel SUTET Putus, 4 Tewas.* Retrieved from: https://tekno.kompas.com/read/2008/02/09/19405840/kabel.sutet.putus.4.tewas. [Indonesian]

Margaretta, D.O., Amalia, N., Utami, F.D., Viridi, S., & Abdullah, M. (2019). Second-order phase transition and universality of self-buckled elastic slender columns. *Journal of Taibah University for Science, 13*, 1128-1136. doi: https://doi.org/10.1080/16583655.2019.1688542

Munir, R., Rahmayanti, H.D., Murniati, R., Rahman, D.Y., Utami, F.D., Viridi, S., & Abdullah, M. (2020). Experiment and modeling of the rice winnowing process: granular segregation method from an ancient era. *Granular Matter, 22(1)*, 24. doi: https://doi.org/10.1007/s10035-019-0991-6

Rahmayanti, H.D., Munir, R., Sustini, E., & Abdullah, M. (2019). Curling evolution of suspended threads replicates 2D self-avoiding walk phenomena and 1D crystallization process. *Journal of Statistical Mechanics: Theory and Experiment*, 2019. 013401. doi: https://doi.org/10.1088/1742-5468/aaf322

Rahmayanti, H.D., F.D. Utami, F.D., & dan M. Abdullah, M. (2016). Physics model for wringing of wet cloth. *European Journal of Physics, 37*, 065806. doi: https://doi.org/10.1088/0143-0807/37/6/065806

Sinebar, W. & Abdullah, M. (2018). Experiment in an aircraft cabin. *The Physics Teacher, 56*, 556. Doi: https://doi.org/10.1119/1.5064573.

Susanto, R. (2018). *Cara mudah belajar lompat tali/skipping/jumping rope*. https://www.youtube.com/watch?v=DzhUza6eBoc&t=87s. [Indonesian]

Tribunnews.com (2012). *Kabel SUTET Menimpa Rumah Warga*. Retrieved from https://www.tribunnews.com/regional/2012/03/21/kabel-sutet-menimpa-rumah-warga. [Indonesian]

Tube, C. (2019). *Jom main tali skipping*. Retrieved from: https://www.youtube.com/watch?v=dBody-5d1HU. [Indonesian]

Wolframalpha.com (2020).

Yuliza, E., Amalia, N., Rahmayanti, H.D., Munir, R., Munir, M.M, Khairurrijal, K., & Abdullah, M. (2018). Stability of granular tunnel. *Granular Matter, 20*, 75. doi:https://doi.org/10.1007/s10035-018-0847-5

.