JPPIPA 10(2) (2024)

Jurnal Penelitian Pendidikan IPA

*Journal of Research in Science Education*

http://jppipa.unram.ac.id/index.php/jppipa/index

# Phase Regularization and Additional Potential in Quantum Systems at the Potential Barrier to Maintain Adiabatic Condition

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Received: December 5, 2023
Revised: January 30, 2024
Accepted: February 25, 2024
Published: February 28, 2024

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DOI: 10.29303/jppipa.v10i2.6393



**Abstract:** This research is theoretical research with a literature study that examines methods to maintain characteristics of electrons when moving in a quantum system at the potential barrier. Attempts to maintain the characteristics of electrons in quantum systems are known as adiabatic. The method used in this research is the fast-forward method. This method was first introduced by Masuda and Nakamura in 2010. The fast-forward method is applied to barrier potential in case of electron energy larger and smaller than the potential barrier. This research focuses on preserving the characteristics of electrons by determining the regularization phase and additional potential of the quantum system at the barrier potential. The wave function solution of the system at the potential barrier is approximated by the Schrödinger equation into three regions for each case. The wave function of each region is regularized to be an adiabatic wave function and an additional term in the form of regularization phase (θ) is obtained. Given the regularized Hamiltonian, the additional potential ($\tilde{V}$) is obtained. The obtained regularization phase and additional potential ensure the quantum system at the potential barrier is in the same state from the initial state to the final state.

**Keywords:** Additional Potential; Adiabatic; Fast Forward; Phase Regularization; Potential Barrier

## Introduction

Since the discovery of the quantum world in the last century, the development of quantum technology has progressed rapidly. Several quantum technologies being developed require speed manipulation to overcome decoherence effects such as quantum computing (Masuda & Nakamura, 2022; Steckmann et al., 2023). However, to control the speed of quantum systems is often very difficult because the properties and characteristics of the particles in the system will change. Therefore, the ability to control quickly and accurately without changing the properties of the system is important for the further development of quantum technology (Kiely et al., 2015; Yu et al., 2018). The development of a method to maintain quantum dynamics without changing the characteristics of the system is known as quantum adiabatic (Hutagalung et al., 2023). According to the adiabatic theorem (Born & Fock, 1928), an adiabatic process occurs when the external parameters of the Hamiltonian are slowly changed (Santos & Sarandy, 2018; Setiawan, Sugihakim, et al., 2023). However, this process takes a long time for the wave function to remain in the same eigenstate as the Hamiltonian slowly changes (Taras et al., 2021).

Adiabatic quantum evolution is a method that has attracted the interest of researchers recently (Setiawan, Ekawita, et al., 2023). In previous research, Masuda and Nakamura have proposed a method that can accelerate the adiabatic quantum dynamics of wave functions in quantum mechanics to obtain the final adiabatic state in a shorter time (Khujakulov & Nakamura, 2016). This method is called the fast-forward method (Setiawan, Gunara, & Nakamura, 2019), and can be defined as creating and rearranging an event on an accelerated timescale like the rapid projection of a film on a screen (Babajanova et al., 2018; Nakamura et al., 2017). Fast forward theory was first introduced by Masuda and

**How to Cite:**
Dari, A., Setiawan, I., & Purwanto, A. (2024). Phase Regularization and Additional Potential in Quantum Systems at the Potential Barrier to Maintain Adiabatic Condition. *Jurnal Penelitian Pendidikan IPA*, *10*(2), 859–869. https://doi.org/10.29303/jppipa.v10i2.6393

Nakamura in 2010. This method applies the additional phase and driving potential determined by the regularized terms (Setiawan et al., 2021) of the wave function to the quantum system (Setiawan, Gunara, Avazbaev, et al., 2019).

The fast-forward method is often related to the shortcuts to adiabaticity (STA) method (Patra & Jarzynski, 2021). Both of these methods are alternatives to adiabatic systems to control the dynamics of classical and quantum systems without having to move for a long time (Chung et al., 2019; Guéry-Odelin et al., 2019; Liu & Kong, 2023). In the quantum context, STA has been applied to several systems (Villazon et al., 2019) including heat engine (Hartmann et al., 2020), quantum harmonic oscillator (Abah & Lutz, 2018; Campo et al., 2014; Dupays et al., 2020), and single spin (Çakmak & Müstecaplıoğlu, 2019). However, the fast-forward method is different from the STA method (Setiawan et al., 2017), which is represented by the regularized Schrödinger equation (Sugihakim et al., 2021).

Quantum systems are concepts used to study the behavior of material and energy at the microscopic level, such as protons, atoms, electrons, and other small particles that cannot be seen directly (Aini et al., 2020). In this case, the concept of adiabatic becomes an important factor, because in controlling microscopic quantum systems, the properties and characteristics of the system need to be maintained (Ainayah et al., 2022). The adiabatic concept in fast forward can be an effective method to accelerate adiabatic dynamics in quantum systems (Setiawan, 2019).

The fast-forward method has been extended to various systems (Elisa et al., 2023). In previous research, the application of the fast-forward method has been carried out to accelerate the adiabatic quantum dynamics of single spin (Benggadinda & Setiawan, 2021). Previous research by (Hutagalung et al., 2023) has also applied fast-forward theory to study adiabatic quantum dynamics by reviewing the case of quantum harmonic oscillators. In another research, fast forward theory is also used to accelerate Dirac particle dynamics (Deffner, 2016), classical adiabatic invariant dynamic construction (Jarzynski et al., 2017), and stochastic heat engine (Nakamura et al., 2020).

In this research, fast forward theory will be applied to quantum systems in the potential barrier. The potential barrier is a potential that restricts the movement of particles in a system (Agustin et al., 2019). In the potential barrier system, there are two cases analyzed, including when the electron energy is larger than the potential barrier and the electron energy is smaller than the potential barrier (Romadani & Rani, 2020; Wardani et al., 2020). The use of the fast-forward method focuses on maintaining the characteristics of electrons in quantum systems at a potential barrier which is carried out by determining the regularization phase and additional potential. Therefore, this research aims to determine the regularization phase and additional potential in quantum systems at a potential barrier.

## Method

The type of research used in this research is a literature study. The literature study was conducted to review physical theories related to quantum adiabatic and the application of the fast-forward method in previous research. This research was conducted with the procedure in Figure 1.

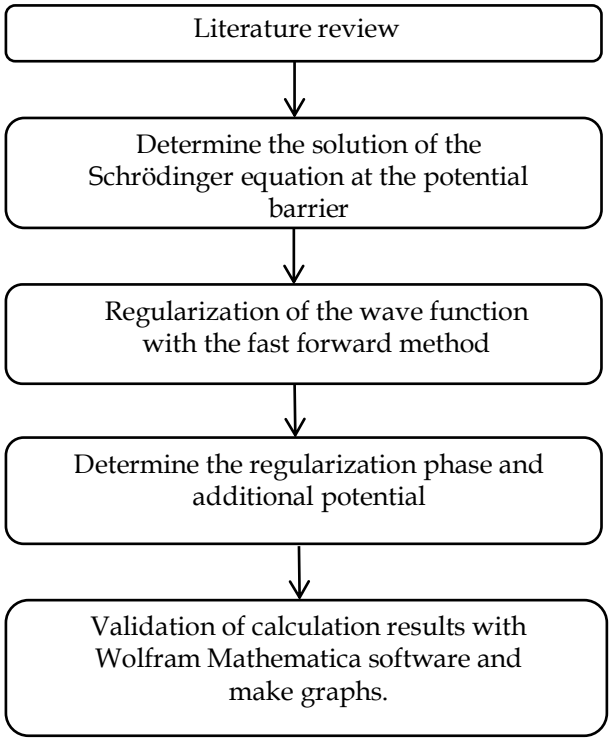


**Figure 1.** The research procedures

The first step in this research is a literature review which is carried out by collecting various reference sources related to fast forward theory, then theoretical studies are carried out by reading and examining fast forward theory and applications of this theory in previous research. The second step is to determine the wave function solution for the barrier potential using the Schrödinger equation. At this step, the wave function solution for the potential barrier was determined based on the literature, but modified to obtain a more comprehensive solution. The wave function solutions are carried out for the case of electron energy larger than the potential barrier and then for the case of electron energy smaller than the potential barrier

In the third step, the wave function obtained at the barrier potential is regularized using the fast-forward method so that it becomes an adiabatic wave function.

In the next step, the regularization phase (θ) and additional potential (Ṽ) of the modified wave function are determined. The regularization phase and additional potential will be determined for each case at the potential barrier. The last step is to validate the calculation results using Wolfram Mathematica software while visualizing the wave function through graphs.

## Result and Discussion

The potential barrier is one type of potential in quantum systems that restricts the movement of particles in the system.

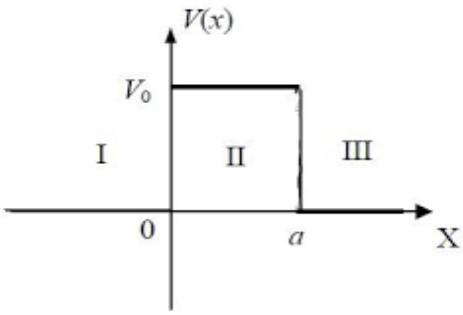


**Figure 2.** The potential barrier

Each case of the potential barrier is divided into three solution areas (Figure 2) with the following potential functions (Zettili, 2009);

$$V(x) = \begin{cases} 0, & x < 0, \\ V_0, & 0 \leq x \leq a, \\ 0, & x > a, \end{cases} \quad (1)$$

where $V_0$ is the potential barrier.

Particle dynamics in quantum systems are represented by the Schrödinger equation. The Schrödinger equation is a second-order differential equation that describes the properties of a particle through wave functions and energy levels (Elviyanti et al., 2023). The general form of the Schrödinger equation can be written as follows:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V\Psi(x,t) \quad (2)$$

where $\Psi(x,t)$ is the wave function that determines the position ($x$) relative to time ($t$), $V$ is the potential, $m$ is the mass of the particle, and $i$ is the square root of -1, and $\hbar$ is reduced Planck's constant. To obtain the solution of the barrier potential system, the time-independent Schrödinger equation is used which is written as follows:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x) \quad (3)$$

where $\psi(x)$ is the time-independent wave function and $E$ is the total energy of the particle.

*Solution for the Case* $E > V_0$

In the case of electron energy is larger than the potential barrier, the electrons have enough energy to pass through the potential barrier. By using equation (3) and the regional boundaries in equation (1), the wave function for each region is obtained as follows:

Region 1 (V = 0)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_1(x)}{dx^2} = E\psi_1(x) \quad (4)$$

by using $k_1 = \frac{\sqrt{2mE}}{\hbar}$ so that the wave function of region 1 can be written as follows:

$$\psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} \quad (5)$$

Region 2 ($V = V_0$)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_2(x)}{dx^2} + V_0\psi_2(x) = E\psi_2(x) \quad (6)$$

by using $k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$ so that the wave function of region 2 can be written as follows:

$$\psi_2(x) = Ce^{ik_2x} + De^{-ik_2x} \quad (7)$$

Region 3 (V = 0)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_3(x)}{dx^2} = E\psi_3(x) \quad (8)$$

by using $k_1 = \frac{\sqrt{2mE}}{\hbar}$ so that the wave function of region 3 can be written as follows:

$$\psi_3(x) = Fe^{ik_1x} + Ge^{-ik_1x} \quad (9)$$

Solution in region 1, the function $\psi_1$ consists of two wave functions moving in $+x$ direction ($e^{ik_1x}$) and $-x$ direction ($e^{-ik_1x}$). In this case, the electron initially moves in the $+x$ direction and then experiences reflection and transmission. Similarly, in region 2, while in region 3 there is only a transmission wave function, so the coefficient G = 0. So, the wave function for each region can be written in the following equation;

$$\psi(x) = \begin{cases} \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} & x < 0, \\ \psi_2(x) = Ce^{ik_2x} + De^{-ik_2x} & 0 \leq x \leq a, \\ \psi_3(x) = Fe^{ik_1x} & x > a, \end{cases} \quad (10)$$

(Sudiarta, 2019).

The constants A, B, C, D, and F can be determined with the following boundary conditions:

$$\psi_1(0) = \psi_2(0), \quad \frac{d\psi_1(0)}{dx} = \frac{d\psi_2(0)}{dx}, \quad (11)$$

$$\psi_2(a) = \psi_3(a), \quad \frac{d\psi_2(0)}{dx} = \frac{d\psi_3(a)}{dx}, \quad (12)$$

The above equation yields the following equation:

$$A + B = C + D \quad (13)$$

$$ik_1(A - B) = ik_2(C - D) \quad (14)$$

$$Ce^{ik_2a} + De^{-ik_2a} = Fe^{ik_1a} \quad (15)$$

$$ik_2(Ce^{ik_2a} - De^{-ik_2a}) = ik_1Fe^{ik_1a} \quad (16)$$

From the four equations above, the constants A, B, C, and D can be obtained, each containing the constant F;

$$A = \frac{(k_2+k_1)^2-(k_2-k_1)^2e^{2ik_2a}}{4k_1k_2e^{-ik_1a}e^{ik_2a}}F \quad (17)$$

$$B = \frac{(k_2^2-k_1^2)(e^{2ik_2a}-1)}{4k_1k_2e^{-ik_1a}e^{ik_2a}}F \quad (18)$$

$$C = \frac{(k_2+k_1)}{2k_2e^{-ik_1a}e^{ik_2a}}F \quad (19)$$

$$D = \frac{(k_2-k_1)e^{2ik_2a}}{2k_2e^{-ik_1a}e^{ik_2a}}F \quad (20)$$

Substituting of equations (17), (18), (19), and (20) into equation (10), for simplify used variables $p = [k_2 + k_1]$, $q = [k_2 - k_1]$, $\alpha = e^{2ik_2a}$ and $\beta = e^{-2ik_2a}$, so that the wave function for each region can be written as follows:

$$\psi_1(x) = \frac{(p^2-q^2\alpha)e^{ik_1x}+pq(\alpha-1)e^{-ik_1x}}{4k_1k_2e^{iaq}}F \quad (21)$$

$$\psi_2(x) = \frac{pe^{ik_2x}+q\alpha e^{-ik_2x}}{2k_2e^{iaq}}F \quad (22)$$

$$\psi_3(x) = Fe^{ik_1x} \quad (23)$$

*Solution for the Case* $E < V_0$

Classically, when the particle has a smaller energy than the barrier then the particle will be reflected all and can't penetrate the barrier wall. However, in quantum systems electrons can break through the barrier because the wave function outside the barrier is not zero. This is known as the tunnelling phenomenon (Wen & Wu, 2020). The solution of the Schrödinger equation for the case $E < V_0$ is almost the same as equation (10), except for the region 2 wave function.

$$\psi(x) = \begin{cases} \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} & x < 0, \\ \psi_2(x) = Ce^{k_2x} + De^{-k_2x} & 0 \le x \le a, \\ \psi_3(x) = Fe^{ik_1x} & x > a, \end{cases} \quad (24)$$

(Sudiarta, 2019).

where $k_1 = \frac{\sqrt{2mE}}{\hbar}$ dan $k_2 = \frac{\sqrt{2m(V_0-E)}}{\hbar}$. Through the same steps as done in the previous case, the following constants A, B, C, and D are obtained, each containing the constant F;

$$A = \frac{(k_2+ik_1)^2-(k_2-ik_1)^2e^{2k_2a}}{4ik_1k_2e^{-ik_1a}e^{k_2a}}F \quad (25)$$

$$B = \frac{(k_2^2+k_1^2)(e^{2k_2a}-1)}{4ik_1k_2e^{-ik_1a}e^{k_2a}}F \quad (26)$$

$$C = \frac{(k_2+ik_1)}{2k_2e^{-ik_1a}e^{k_2a}}F \quad (27)$$

$$D = \frac{(k_2-ik_1)e^{2k_2a}}{2k_2e^{-ik_1a}e^{k_2a}}F \quad (28)$$

Substituting of equations (25), (26), (27), and (28) into equation (24), for simplify used variables $p' = [k_2 + ik_1]$, $q' = [k_2 - ik_1]$, $\alpha' = e^{2k_2a}$ and $\beta' = e^{-2k_2a}$ into constants A, B, C, and D so that the wave function for each region can be written as follows:

$$\psi_1(x) = \frac{(p'^2-q'^2\alpha')e^{ik_1x}+p'q'(\alpha'-1)e^{-ik_1x}}{4ik_1k_2e^{q'a}}F \quad (29)$$

$$\psi_2(x) = \frac{p'e^{k_2x}+q'\alpha'e^{-k_2x}}{2k_2e^{q'a}}F \quad (30)$$

$$\psi_3(x) = Fe^{ik_1x} \quad (31)$$

After obtaining the wave functions in all three regions at the potential barrier for the case of electron energy larger ($E > V_0$) and smaller ($E < V_0$) than the potential barrier, then through the fast forward method the wave functions will be regularized to become adiabatic wave functions that will be used to obtain the regularization phase $\theta$ and the additional potential $\tilde{V}$.

In general, the wave function solution of the Schrödinger equation at the potential barrier can be written as follows:

$$\Psi(x,t) = \psi_n e^{-i\omega t} \quad (32)$$

where $n = 1,2,3$ which is the region of the wave function solution at the potential barrier, and $\omega$ is angular frequency of the wave. By modifying ($\omega = \frac{E}{\hbar}$), equation (32) can be rewritten into the following equation:

$$\Psi(x,t) = \psi_n e^{-\frac{i}{\hbar}\int_0^t E_n dt} \quad (33)$$

In order for the wave function in equation (33) to be an adiabatic wave function, the parameter $t$ is modified to $R(t)$, with:

$$R(t) = R_0 + \varepsilon t \; ; \; \frac{dR(t)}{dt} = \varepsilon \quad (34)$$

where $\varepsilon \to 0$ is an adiabatic parameter that is very small and causes the system to move slowly. If the system is initially in an eigenstate at the $n$th energy, then the adiabatic theorem guarantees that in the limit $\varepsilon \to 0$ the wave function will remain in the same eigenstate. By adding the adiabatic parameter ε and the regularization phase $\theta$, the regularized wave function can be written as follows:

$$\Psi^{(reg)}\big(x,R(t)\big) = \psi_n(x,R(t))e^{-(i/\hbar)\int_0^t E_n(R(t))\,dt}e^{i\varepsilon\theta(x,t)} \quad (35)$$

The Schrödinger equation for $\Psi^{(\mathrm{reg})}$ is represented as follows:

$$i\hbar\frac{\partial\Psi^{(reg)}(x,R(t))}{\partial t} = -\frac{\hbar^2}{2m_0}\frac{\partial^2\Psi^{(reg)}(x,R(t))}{\partial x^2} + V_0{}^{(reg)}(x,R(t))\Psi^{(reg)}(x,R(t)) \quad (36)$$

where $V_0{}^{(reg)}$ is the regularized potential obtained from the regularized Hamiltonian when regularizing the wave function. $V_0{}^{(reg)}$ can be written as follows:

$$V_0{}^{(reg)}(x,t) = V_0\big(x,R(t)\big) + \varepsilon\tilde{V}(x,t) \quad (37)$$

$\tilde{V}$ is the additional potential of the Hamiltonian regularisation condition by (Masuda & Nakamura, 2010). Then substitute equations (35) and (37) into equation (36), yields:

$$i\hbar\varepsilon\frac{\partial\psi_n}{\partial R} + E_n\psi_n - \hbar\varepsilon\frac{\partial\theta}{\partial t}\psi_n = -\frac{\hbar^2}{2m_0}\left[\frac{\partial^2\psi_n}{\partial x^2} + 2i\varepsilon\frac{\partial\psi_n}{\partial x}\frac{\partial\theta}{\partial x} + \psi_n i\varepsilon\frac{\partial^2\theta}{\partial x^2} - \psi_n\varepsilon^2\left(\frac{\partial\theta}{\partial x}\right)^2\right] + V_0\psi_n + \varepsilon\tilde{V}(x,t)\psi_n \quad (38)$$

Substituting of the time-independent Schrödinger equation in equation (3) into equation (38), yields:

$$i\hbar\varepsilon\frac{\partial\psi_n}{\partial R} - \hbar\varepsilon\frac{\partial\theta}{\partial t}\psi_n = -\frac{\hbar^2}{2m_0}\left[2i\varepsilon\frac{\partial\psi_n}{\partial x}\frac{\partial\theta}{\partial x} + \psi_n i\varepsilon\frac{\partial^2\theta}{\partial x^2} - \varepsilon^2\left(\frac{\partial\theta}{\partial x}\right)^2\psi_n\right] + \varepsilon\tilde{V}(x,t)\psi_n \quad (39)$$

Eliminate $\varepsilon$ in both sides and eliminate the equation containing $\varepsilon^2$ because $\varepsilon$ is very small, so equation (39) can be rewritten as:

$$i\hbar\frac{\partial\psi_n}{\partial R} - \hbar\varepsilon\frac{\partial\theta}{\partial t}\psi_n = -\frac{\hbar^2}{2m_0}\left[2i\frac{\partial\psi_n}{\partial x}\frac{d\theta}{dx} + \psi_n i\frac{\partial^2\theta}{\partial x^2}\right] + \tilde{V}(x,t)\psi_n \quad (40)$$

Then multiply both sides of equation (40) by $(i/\hbar)\psi_n^*$, then separate the real and imaginary parts as follows;

$$|\psi_n|^2\frac{\partial^2\theta}{\partial x^2} + 2Re\left[\psi_n\frac{\partial\psi_n^*}{\partial x}\right]\frac{\partial\theta}{\partial x} + \frac{2m_0}{\hbar}Re\left[\psi_n\frac{\partial\psi_n^*}{\partial R}\right] = 0 \quad (41)$$

and

$$\frac{\hbar}{m_0}\mathrm{Im}\left(\psi_n^*\frac{\partial\psi_n}{\partial x}\right)\frac{\partial\theta}{\partial x} + \frac{\tilde{V}}{\hbar}|\psi_n|^2 + \mathrm{Im}\left[\psi_n^*\frac{\partial\psi_n}{\partial R}\right] + \frac{\partial\theta}{\partial t}|\psi_n|^2 = 0 \quad (42)$$

(Masuda & Nakamura, 2010).

Use the equation (41) to determine the regularization phase $\theta$. Since $\psi_n$ is independent of $R$, equation (41) becomes:

$$|\psi_n|^2\frac{\partial^2\theta}{\partial x^2} + 2Re\left[\psi_n\frac{\partial\psi_n^*}{\partial x}\right]\frac{\partial\theta}{\partial x} = 0 \quad (43)$$

The wave function at the potential barrier is not composed of real functions, so equation (51) can be written as:

$$|\psi_n|^2\frac{\partial^2\theta}{\partial x^2} = 0 \quad (44)$$

$$\theta = \frac{C_0\,x}{|\psi_n|^2} \quad (45)$$

where $C_0$ is a constant of the integral to obtain regularization phase $\theta$. Furthermore, equation (42) is used to obtain the additional potential;

$$\tilde{V} = -\hbar\,\mathrm{Im}\left[\frac{\partial\psi_n}{\partial R}\frac{1}{\psi_n}\right] - \frac{\hbar^2}{m_0}Im\left[\frac{1}{\psi_n}\frac{\partial\psi_n}{\partial x}\right]\frac{\partial\theta}{\partial x} \quad (46)$$

$$\tilde{V} = -\frac{\hbar^2}{m_0}\mathrm{Im}\left[\frac{1}{\psi_n}\frac{\partial\psi_n}{\partial x}\right]\frac{\partial\theta}{\partial x} \quad (47)$$

For the case $E > V_0$, use the equation (45) and the wave functions in the equation (21), (22), and (23) to obtain the regularization phase and additional potential for each region;

*Regularization Phase for the Case $E > V_0$*

$$\theta_1 = \frac{16{k_1}^2{k_2}^2C_0\,x}{F^2[(p^2-q^2\beta)e^{-2ik_1x}+pq(\beta-1)][(p^2-q^2\alpha)e^{2ik_1x}+pq(\alpha-1)]} \quad (48)$$

$$\theta_2 = \frac{4{k_2}^2C_0\,x}{F^2\,[p^2+q^2+pq(\alpha e^{-2ik_2x}+\beta e^{2ik_2x})]} \quad (49)$$

$$\theta_3 = \frac{C_0x}{F^2} \quad (50)$$

Using the equation (47), additional potential for each region is obtained as follows:

*Additional Potential for the Case $E > V_0$*

$$\tilde{V}_1 = -\frac{16i\hbar^2{k_1}^3{k_2}^2C_0\left((p^2-q^2\alpha)e^{2ik_1x} - pq(\alpha-1)\right)}{F^2m_0\left((p^2-q^2\beta)e^{-2ik_1x}+pq(\beta-1)\right)^2} \frac{\left(\begin{matrix}(p^2+q^2)^2 - 2p^2q^2(\alpha+\beta)+pq(\alpha-1)(p^2-q^2\beta)(1+2ik_1x)e^{-2ik_1x}+\\ pqe^{2ik_1x}(\beta-1)(p^2-q^2\alpha)(1-2ik_1x)\end{matrix}\right)}{\left((p^2-q^2\alpha)e^{2ik_1x}+pq(\alpha-1)\right)^3} \quad (51)$$

$$\tilde{V}_2 = -\frac{4i\hbar^2{k_2}^3C_0(pe^{2ik_2x} - q\alpha)}{F^2m_0(pe^{-2ik_2x}+q\beta)^2}\frac{\left(\begin{matrix}p^2+q^2+pq\alpha e^{-2ik_2x}(1+2ik_2x)\\ +pq\beta e^{2ik_2x}(1-2ik_2x)\end{matrix}\right)}{(pe^{2ik_2x}+q\alpha)^3} \quad (52)$$

$$\tilde{V}_3 = -\frac{i\hbar^2k_1C_0}{F^2m_0} \quad (53)$$

After obtaining the regularization phase and additional potential, the regularized wave function for each region can be written as follows:

$$\Psi_1^{(reg)} = \frac{(p^2-q^2\alpha)e^{ik_1x}+pq(\alpha-1)e^{-ik_1x}}{4k_1k_2e^{iaq}} Fe^{-\left(\frac{i}{\hbar}\right)\int_0^t E_n(R(t))\,dt}$$
$$e^{\frac{i\varepsilon 16{k_1}^2{k_2}^2C_0\,x}{F^2\left[(p^2-q^2\beta)e^{-2ik_1x}+pq(\beta-1)\right]\left[(p^2-q^2\alpha)e^{2ik_1x}+pq(\alpha-1)\right]}} \quad (54)$$

The electron distribution in region 1 ($x < 0$) can be represented through the probability density of the regulated wave function $\left|\Psi_1^{(\mathrm{reg})}\right|^2$ on the graphs in Figure 3 for 2D and Figure 4 for 3D.

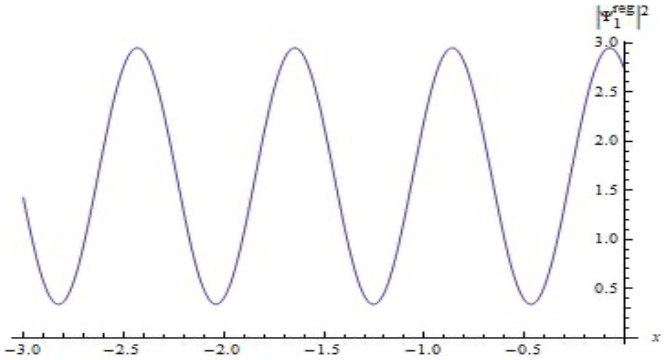


**Figure 3**. 2D graph of probability density of wave function of region 1 ($\mathbf{E > V_0}$) for $\boldsymbol{x \leq 0}$ using Wolfram Mathematica

Figure 3 shows graph 2D of the probability density of the wave function in region 1 of the case E > $V_0$ for $x = -3$ to $x = 0$, $k_1 = 4$, $k_2 = 2$, $t = 1$, $F = 1$ and $a = 2$. Probability density represents the presence of electrons in the system. Although the wave function is regularized with an additional term in the form of a regularization phase, it will not change its probability density. In the case that the energy of the electron is greater than its barrier potential, the wave function has a sufficiently large amplitude in the first region.

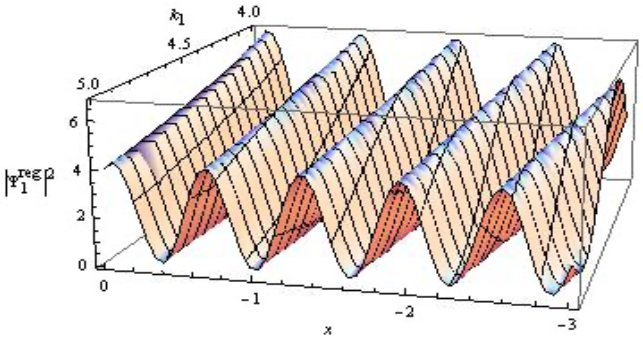


**Figure 4**. 3D graph of probability density of wave function of region 1 ($\mathbf{E > V_0}$) for $\boldsymbol{x \leq 0}$ using Wolfram Mathematica

Figure 4 shows graph 3D of the probability density of the wave function in region 1 (E > $V_0$) for $x = -3$ to $x = 0$, $k_1 = 4$ to $k_1 = 5$, $k_2 = 2$, $t = 1$, $F = 1$ dan $a = 2$. The graph above shows that when $k_1$ is magnified the wave frequency will be higher.

$$\Psi_2^{(reg)} = \frac{pe^{ik_2x}+q\alpha e^{-ik_2x}}{2k_2e^{iaq}} Fe^{-\left(\frac{i}{\hbar}\right)\int_0^t E_n(R(t))\,dt}$$
$$e^{\frac{i\varepsilon 4{k_2}^2C_0\,x}{F^2\left[p^2+q^2+pq\left(\alpha e^{-2ik_2x}+\beta e^{2ik_2x}\right)\right]}} \quad (55)$$

The electron distribution in region 2 ($0 \leq x \leq a$) can be represented through the probability density of the regulated wave function $\left|\Psi_2^{(\mathrm{reg})}\right|^2$ on the graphs in Figure 5 for 2D and Figure 6 for 3D.

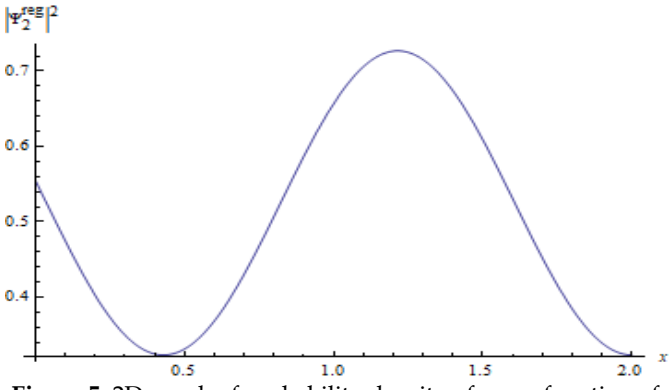


**Figure 5.** 2D graph of probability density of wave function of region 2 ($\mathbf{E > V_0}$) for $\boldsymbol{0 \leq x \leq a}$ using Wolfram Mathematica

Figure 5 shows graph 2D of the probability density of the wave function in region 2 of the case E > $V_0$ for $x = 0$ to $x = 2$, $k_1 = 4$, $k_2 = 2$, $t = 1$, $F = 1$ and $a = 2$.

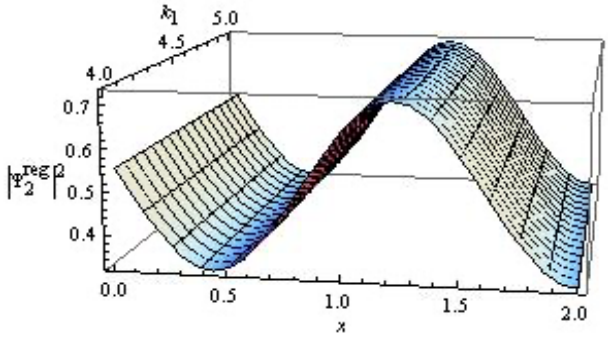


**Figure 6.** 3D graph of probability density of wave function of region 2 ($\mathbf{E > V_0}$) for $\boldsymbol{0 \leq x \leq a}$ using Wolfram Mathematica

Figure 6 shows graph 3D of the probability density of the wave function in region 2 (E > $V_0$) for $x = 0$ to $x = 2$, $k_1 = 4$ to $k_1 = 5$, $k_2 = 2$, $t = 1$, $F = 1$ dan $a = 2$. Based on figure 5 and figure 6, it can be seen that after passing through the barrier, the wave function in region 2 has a lower amplitude so that the frequency of the wave decreases than region 1.

$$\Psi_3^{(reg)} = Fe^{ik_1x}e^{-\left(\frac{i}{\hbar}\right)\int_0^t E_n(R(t))\,dt}e^{\frac{i\varepsilon C_0\,x}{F^2}} \quad (56)$$

The electron distribution in region 3 ($x > a$) can be represented through the probability density of the regulated wave function $\left|\Psi_3^{(\mathrm{reg})}\right|^2$ on the graphs in Figure 7 for 2D and Figure 8 for 3D.

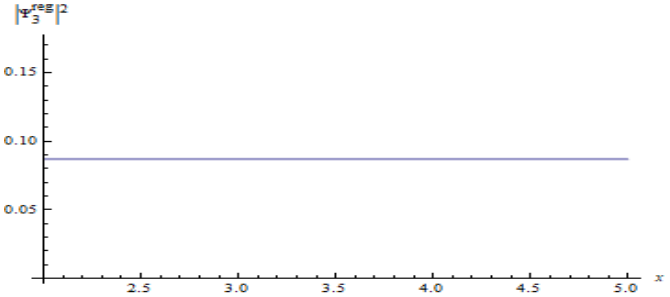


**Figure 7.** 2D graph of probability density of wave function of region 3 ($\mathbf{E} > \mathbf{V_0}$) for $\boldsymbol{x \geq a}$ using Wolfram Mathematica

Figure 7 shows graph 2D of the probability density of the wave function in region 3 of the case E > $V_0$ for $x = 2$ to $x = 5$, $k_1 = 4$, $k_2 = 2$, $t = 1$, $F = 1$ and $a = 2$.

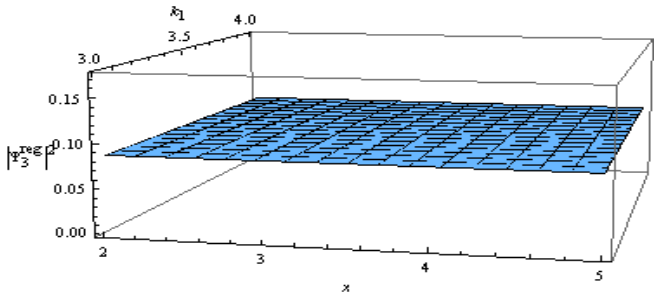


**Figure 8.** 3D graph of probability density of wave function of region 3 ($\mathbf{E} > \mathbf{V_0}$) for $\boldsymbol{x \geq a}$ using Wolfram Mathematica

Figure 8 shows graph 3D of the probability density of the wave function in region 3 (E > $V_0$) for $x = 2$ to $x = 5$, $k_1 = 3$ to $k_1 = 4$, $k_2 = 2$, $t = 1$, $F = 1$ dan $a = 2$. In region 3, the wave function experiences total transmission to the right because there is no barrier in region 3 so that the consistency of the medium through which the wave passes causes the wave to travel straight.

For the case E < $V_0$ use the equation (45) and wave functions in the equation (29), (30), and (31) to obtain the regularization phase and additional potential for each region;

*Regularization Phase for the Case* $E < V_0$

$$\theta_1 = \frac{-16\alpha' {k_1}^2 {k_2}^2 C_0\, x}{F^2\left[(q'^2-p'^2\alpha')e^{-2ik_1x}+p'q'(\alpha'-1)\right]\left[(p'^2-q'^2\alpha')e^{2ik_1x}+q'p'(\alpha'-1)\right]} \quad (57)$$

$$\theta_2 = \frac{4{k_2}^2 C_0\, x}{F^2\left[p'^2+q'^2+p'q'\left(\alpha e^{-2k_2x}+\beta e^{2k_2x}\right)\right]} \quad (58)$$

$$\theta_3 = \frac{C_0\, x}{F^2} \quad (59)$$

Using the equation (47), additional potential for each region is obtained as follows:

*Additional Potential for the Case* $E < V_0$

$$\tilde{V}_1 = \frac{16i\hbar^2 {k_1}^3 {k_2}^2 C_0\left((p'^2-q'^2\alpha')e^{2ik_1x}-p'q'(\alpha'-1)\right)}{F^2 m_0\left((q'^2-p'^2\alpha')e^{-2ik_1x}+p'q'(\alpha'-1)\right)^2} \frac{\left(\begin{array}{c}2p'^2q'^2(\alpha'+\beta')-\left(p'^2+q'^2\right)^2+p'q'(1-\beta') \\ \left((q'^2-p'^2\alpha')(1+2ik_1x)e^{-2ik_1x}+(p'^2-q'^2\alpha')(1-2ik_1x)e^{2ik_1x}\right)\end{array}\right)}{\left((p'^2-q'^2\alpha')e^{2ik_1x}+q'p'(\alpha'-1)\right)^3} \quad (60)$$

$$\tilde{V}_2 = 0 \quad (61)$$

$$\tilde{V}_3 = -\frac{i\hbar^2 k_1 C_0}{F^2 m_0} \quad (62)$$

After obtaining the regularization phase and additional potential, the regularized wave function for each region can be written as follows:

$$\Psi_1{}^{(reg)} = \frac{(p'^2-q'^2\alpha')e^{ik_1x}+p'q'(\alpha'-1)e^{-ik_1x}}{4k_1k_2e^{q'a}}F \; e^{-\left(\frac{i}{\hbar}\right)\int_0^t E_n(R(t))\,dt} \; e^{\frac{i\varepsilon 16\alpha' {k_1}^2 {k_2}^2 C_0\, x}{F^2\left[(q'^2-p'^2\alpha')e^{-2ik_1x}+p'q'(\alpha'-1)\right]\left[(p'^2-q'^2\alpha')e^{2ik_1x}+q'p'(\alpha'-1)\right]}} \quad (63)$$

The electron distribution in region 1 ($x < 0$) can be represented through the probability density of the regulated wave function $\left|\Psi_1{}^{(\mathrm{reg})}\right|^2$ on the graph in Figure 9 for 2D and Figure 10 for 3D.

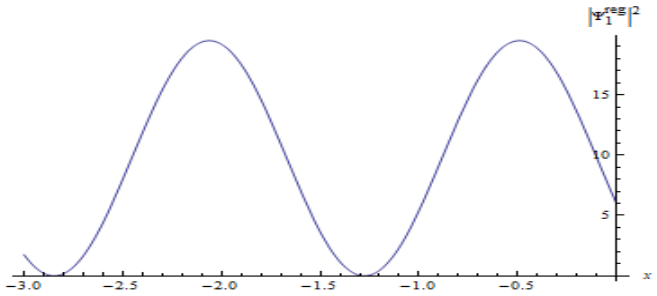


**Figure 9.** 2D graph of probability density of wave function of region 1 ($\mathbf{E} < \mathbf{V_0}$) for $\boldsymbol{x \leq 0}$ using Wolfram Mathematica

Figure 9 shows graph 2D of the probability density of the wave function in region 1 of the case E < $V_0$ for $x = -3$ to $x = 0$, $k_1 = 2$, $k_2 = 3$, $t = 1$, $F = 1$ and $a = 2$.

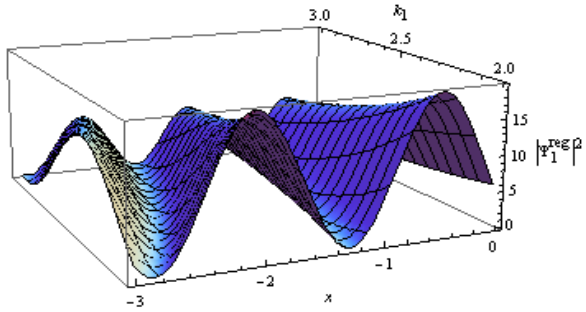


**Figure 10.** 3D graph of probability density of wave function of region 1 ($\mathbf{E} < \mathbf{V_0}$) for $\boldsymbol{x \leq 0}$ using Wolfram Mathematica

Figure 10 shows graph 3D of the probability density of the wave function in region 1 of the case $E < V_0$ for $x = -3$ to $x = 0$, $k_1 = 2$ to $k_1 = 3$ , $k_2 = 3$, $t = 1$, $F = 1$ and $a = 2$. In this case, if $k_2$ is increased, the potential barrier will be even greater. In this region the electrons are traveling towards the barrier potential with smaller energy than the potential of the barrier. Classically, electrons will reflect and nothing is transmitted. But in a quantum system, electrons will still experience transmission called quantum tunneling.

$$\Psi_2^{(reg)} = \frac{p_I e^{k_2 x} + q_I \alpha_I e^{-k_2 x}}{2k_2 e^{q_I a}} F e^{-\left(\frac{i}{\hbar}\right)\int_0^t E_n(R(t))\,dt} e^{\frac{i\varepsilon 4{k_2}^2 C_0 x}{F^2\left[p_I^2 + q_I^2 + p_I q_I\left(\alpha e^{-2k_2 x} + \beta e^{2k_2 x}\right)\right]}} \quad (64)$$

The electron distribution in region 2 ($0 \le x \le a$) can be represented through the probability density of the regulated wave function $\left|\Psi_2^{(\mathrm{reg})}\right|^2$ on the graph in Figure 11 for 2D and Figure 12 for 3D.

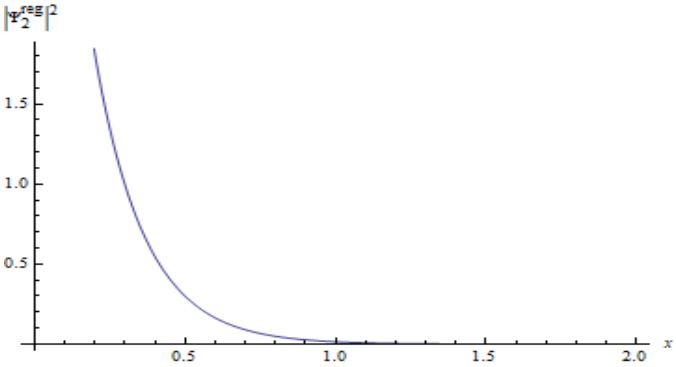


**Figure 11.**2D graph of probability density of wave function of region 2 ($\mathbf{E < V_0}$) for $\boldsymbol{0 \le x \le a}$ using Wolfram Mathematica

Figure 11 shows graph 2D of the probability density of the wave function in region 2 of the case $E < V_0$ for $x = 0$ to $x = 2$, $k_1 = 2$, $k_2 = 3$, $t = 1$, $F = 1$ and $a = 2$.

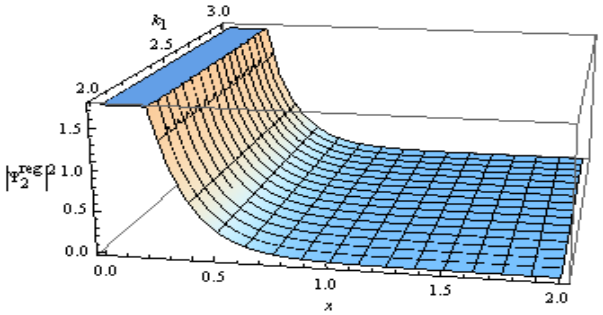


**Figure 12.**2D graph of probability density of wave function of region 2 ($\mathbf{E < V_0}$) for $\boldsymbol{0 \le x \le a}$ using Wolfram Mathematica

Figure 12 shows graph 3D of the probability density of the wave function in region 2 of the case $E < V_0$ for $x = 0$ to $x = 2$, $k_1 = 2$ to $k_1 = 3$ , $k_2 = 3$, $t = 1$, $F = 1$ and $a = 2$. Based on Figure 11 and Figure 12 on the graph, it can be seen that the electrons have passed through the potential barrier and experienced a decrease in energy so that the amplitude decreases.

$$\Psi_3^{(reg)} = F e^{ik_1 x} e^{-\left(\frac{i}{\hbar}\right)\int_0^t E_n(R(t))\,dt} e^{\frac{C_0 x}{F^2}} \quad (65)$$

The electron distribution in region 3 ($x > a$) can be represented through the probability density of the regulated wave function $\left|\Psi_3^{(\mathrm{reg})}\right|^2$ on the graph in Figure 13 for 2D and Figure 14 for 3D.

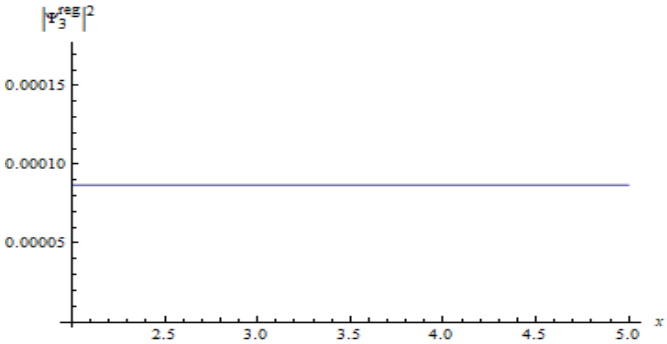


**Figure 13.** 2D graph of probability density of wave function of region 3 ($\mathbf{E < V_0}$) for $\boldsymbol{x \ge a}$ using Wolfram Mathematica

Figure 13 shows graph 2D of the probability density of the wave function in region 3 of the case $E < V_0$ for $x = 2$ to $x = 5$, $k_1 = 2$, $k_2 = 3$, $t = 1$, $F = 1$ and $a = 2$.

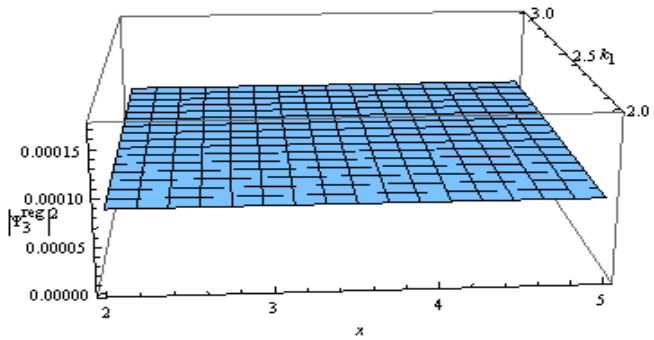


**Figure 14.** 3D graph of probability density of wave function of region 3 ($\mathbf{E > V_0}$) for $\boldsymbol{x \ge a}$ using Wolfram Mathematica

Figure 12 shows graph 3D of the probability density of the wave function in region 3 of the case $E < V_0$ for $x = 2$ to $x = 5$, $k_1 = 2$ to $k_1 = 3$ , $k_2 = 3$, $t = 1$, $F = 1$ and $a = 2$. In region 3 there is no barrier that causes the electrons to reflect so the electrons will experience total transmission to the right as in the previous case.

The regularization phase and additional potential that have been obtained in the case of electron energy larger and smaller than the potential barrier for each region can be used to maintain the adiabatic state of electrons in the quantum system at the potential barrier even though the system is driven in a shorter time. To accelerate the adiabatic quantum dynamics in this

system can be continued by determining the driving potential using the fast-forward method. The existence of the regularization phase and additional potential in the wave function will maintain the adiabatic state of the system when the control of the system is carried out, the characteristics of the particles in the system will not change.

## Conclusion

Phase regularization and additional potential in quantum systems at potential barriers have been obtained. In the case of electron energy larger than the potential barrier ($E > V_0$), the regularization phase and the additional potential are shown in equations (48) and (51) for region 1, equations (49) and (52) for region 2, and equations (50) and (53) for region 3, while in the case of electron energy smaller than the potential barrier ($E > V_0$) the regularization phase and the additional potential are shown in equations (57) and (60) for region 1, equations (58) and (61) for region 2, and equations (59) and (62) for region 3. Based on the results, the regularization phase in each region changes as the amplitude changes, as does the additional potential. The regularization phase and the required additional potential are larger to maintain electron characteristics in regions with smaller amplitude. The regularization phase and additional potential obtained will ensure the wave function will remain in the adiabatic state from the initial state to the final state.

**Acknowledgments**
The author would like to thank the Physics Education Study Program, Faculty of Teacher Training and Education (FKIP) Universitas Bengkulu for granting permission to take part in the MBKM (Merdeka Belajar Kampus Merdeka) Research program where this article is one of the outputs.

**Author Contributions**
Conceptualization, Anggen Dari and Iwan Setiawan; methodology, Anggen Dari; validation, Iwan Setiawan and Andik Purwanto; resources, Anggen Dari; data curation, Iwan Setiawan.; writing and editing, Anggen Dari; visualization, Anggen Dari; supervision, Iwan Setiawan, and Andik Purwanto. All authors have read and agree to the published version of the manuscript.

**Funding**
This research received no external funding.

**Conflicts of Interest**
The authors declare no conflict of interest.

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