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Probability of He+ Ion at Quantum Number $3 \leq n \leq 4$ in Momentum Space

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Abstract: Ions resulting from the ionization of helium atoms are known as helium ions. Atom of Helium The probability distribution of the particle locations and velocities inside a helium atom is referred to as probability. Using theoretical study methods, the goal of this research is to find the probability of an electron's position in a helium atom at the quantum number 3 ≤ n ≤ 4. This type of research is non-experimental research by developing previously existing theories. This type of research is non-experimental research by transforming the radial wave function of hydrogenic atoms in position space into a radial wave function in momentum space using the Fourier transformation, then including a number of Gegenbauer functions and using the probability equation of finding an electron in a Helium ion in momentum space. The results of this research provide information regarding the position and probability of the existence of electrons in the helium atom. The probability value for the Helium ion is obtained using the equation $\mathbf{P}_{(p)} = \int_0^p p^2 \left|F_{n,l}(P)^2\right| dp$ which is used to indicate the probability of finding an electron in a helium atom is directly proportional to the principal quantum number (n) and the value of the electron's momentum. The larger the electron momentum interval, the greater the probability.

Keywords: Helium atom; Probability; Radial waves

Introduction

The development of quantum theory around 1900, or toward the close of the 19th century (Syahrial et al., 2022). The physical science of quantum physics examines atoms, subatomic particles, and atomic systems (Sutopo, 2005). The development of quantum physics in atoms provides great knowledge for understanding the universe involving atomic phenomena (Supriadi, Mardhiana, Kristiawan, Kamalia, & Sari, 2023). In the study of atomic physics, molecular physics, particle physics, nuclear physics, and quantum chemistry, mathematical theorems and equations are crucial for building a particle wave function (Plotnitsky, 2020).

The particle wave function is the solution of a particle wave equation, namely a second order partial differential equation based on (i) the law of conservation of energy; (ii) de Broglie's hypothesis; and (iii) convergent and have continuity properties (Setiyowati et al., 2021). A single electron atom can describe the link between waves and particles (Supriadi, Lorensia, Shahira, Prabandari, & Putri, 2023). Because it only has one orbital electron and one proton in its nucleus, the hydrogen atom is the most basic type of atom (Suyanta, 2019). The advancement of quantum mechanical ideas about atoms offers valuable insights into the cosmos concerning atomic processes (Supriadi, Lorensia, Shahira, Prabandari, & Putri, 2023).

In 1926, Erwin Schrödinger succeeded in formulating the wave-particle equation which explains the movement of electrons in atoms. Erwin Schrödinger's discovery of second-order differential equations provided details about the wave behavior of particles, including wave functions and their probabilities (Hakim et al., 2016). This was also

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mentioned by Utami et al. (2019), who came to the conclusion that the Schrodinger equation can be utilized to ascertain an atom's properties and that varying the quantum number can result in a distinct wave function. The wave function (ψ) has no physical meaning, even though it can provide information about a system's condition at any given moment. On the other hand, when the wave function's absolute value is squared and integrated with specific variables, this can yield probability values that indicate the physical interpretation. In classical physics, the Schrodinger equation is equivalent to Newton's second law, which is represented by the formula ($\frac{\partial^2 r}{\partial t^2} = \frac{F}{m}$). The Schrodinger equation is generally expressed as follows:

$$\nabla^2\Psi_{(r,t)} + \frac{2\mu}{\hbar^2}(E-V)\Psi_{(r,t)} = 0 \tag{1}$$

With $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$ in Cartesian coordinates $= \frac{1}{r^2}\left\{\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right) + \frac{1}{sin^2\theta}\frac{d^2}{d\emptyset}\right\}$ in spherical coordinates (Supriadi et al., 2022). So, in spherical coordinates, the steady (time independent) Schrodinger equation of electrons in a hydrogenic atom becomes:

$$\frac{1}{r^2}\left\{\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right) + \frac{1}{sin^2\theta}\frac{d^2}{d\emptyset}\right\}\Psi_{(r,\theta,\phi)} + \frac{2\mu}{\hbar^2}(E-V)\Psi_{(r,\theta,\phi)} = 0 \tag{2}$$

Solving equation (2) can be done by changing the partial differential equation into a simpler differential equation by assuming that $\Psi_{(r,\theta,\phi)} = R(r)Y(\theta,\phi)$. Using the variable separation method, we get:

a. Radial Schrodinger Equation:

$$\left[\frac{1}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right) + \frac{2\mu r^2}{\hbar^2}[E-V(r)]\right] = \beta \tag{3}$$

b. Angular Schrodinger Equation:

$$\left[\frac{1}{Y(\theta,\phi)sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial Y(\theta,\phi)}{\partial\theta}\right) + \frac{1}{Y(\theta,\phi)sin^2\theta}\frac{\partial^2 Y(\theta,\phi)}{\partial\phi^2}\right] = -\beta \tag{4}$$

Equation (3) radial Schrodinger equation and the related Laguerre differential equation are comparable. From a mathematical formulation:

$$L_n^k(r) = \frac{d^k}{dr^k}L_n(r) \tag{5}$$

where $L_n(r)$ is the Laguerre function (Supriadi et al., 2022). The Laguerre function and the Laguerre polynomial are the two primary ideas that are discussed in the Laguerre equation (Kharismawati et al., 2021). Laguerre polynomials and the Laguerre function have numerous uses in theoretical physics and mathematics, including the solution of differential equations, numerical analysis, and other applied scientific domains (Marlina et al., 2022). In mathematical physics, the Laguerre function is a solution to a second-order differential equation called the Laguerre differential equation. It is commonly used in oscillation and central potential problems, among other situations (Dai et al., 2015). By using the associated Laguerre function, the solution of equation (3) can be expressed in terms of the radial function of the hydrogenic atom, namely:

$$R_{nl}(r) = N_{nl}\, e^{-\frac{zr}{na_0}}\left(\frac{2zr}{na_0}\right)^l L_{n+1}^{2l+1}\left(\frac{2zr}{na_0}\right) \tag{6}$$

with $N_{nl} = -\left(\frac{2z}{na_0}\right)^{3/2}\sqrt{\frac{(n-l-1)!}{2n[(n+l)!]^3}}$ is the normalization constant, $L_{n-l-1}^{2l+1}\left(\frac{2Zr}{na_0}\right)$ is the Laguerre polynomial, $a_0 = 0{,}529\times10^{-10}$ meter is the Bohr radius, z is the atomic number, n is the principal quantum number and l is the orbital quantum number (Pandu et al., 2021).

The radial wave function of hydrogenic atoms can not only be expressed in position space, R_nl (r), $R_{nl}(r)$ but can also be expressed in momentum space, $R_{nl}(p)$ (Ningrum et al., 2023). According to Al Bawani et al. (2023), the radial function can be used to see how electrons behave in momentum space Lithium ion electrons' average electron location (anticipated value), or Li^{2+}, in momentum space is dependent on both the momentum interval and the principal quantum number. Hage-Hassan states that the Fourier transform is used to transform the tritium atom's radial wave function in order to obtain the atom's radial wave function in momentum space (Supriadi, Sari, Rahmawati, Mardhiana, & Firdausyiah, 2023). Lithium ions' wave functions in momentum space often don't differ significantly from those of other hydrogenic atoms (Supriadi, Lorensia, Shahira, Prabandari, & Putri, 2023). The radial wave function of hydrogenic atoms in momentum space is formulated (Bransden et al., 1995):

$$F_{nl} = \left(\frac{2}{\pi}\frac{(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}} n^2 2^{2l+2} l! \frac{n^l p^l}{(n^2p^2+1)^{l+2}}\ C_{n-l-1}^{l+1}\left(\frac{n^2p^2-1}{n^2p^2+1}\right) \tag{7}$$

(Nurroniah et al., 2023)

An atom of helium that loses one electron results in an ion with a one-electron orbital, known as a helium ion (He+) (Lutfin et al., 2020). When an electron is released during ionization, helium atoms become helium ions ($^4_2He^+$), leaving them with just one orbital electron.

So the helium ion behaves the same as hydrogen, the difference is that the nucleus has a positive charge of

2e. Helium ions are used in daily life for two purposes: to fill hot air balloons and to facilitate underwater diving on the bottom by substituting normal air (Nurroniah et al., 2023). Helium serves as a semiconductor magnet in applications such as nuclear magnet resonance (NMR) (Alimah et al., 2017) and magnetic resonance imaging (MRI) in its liquid state (Rillo et al., 2015). According to Saputro (2019), the ability of helium ions to destroy bacteria on mangosteen fruit makes them suitable for use as a plasma-based sterilizing agent. Helium gas has the potential to prevent myocardial ischemia and be utilized as an additional therapy for respiratory disorders (Berganza et al., 2013).

According to Max Born, the likelihood of discovering an electron in a substance can be determined by calculating the squared area of the radial wave function (Yanuarief et al., 2019). According to Supriadi et al. (2023) came to the conclusion that the chance of locating an electron in an atom is described by the integral of the absolute value of the radial function, and that the value decreases as the electron's position becomes farther from the atomic nucleus. Probability is the chance of finding an electron in an atom or ion. Then the probability of getting an electron in a hydrogenic atom (Helium ion) is (Supriadi, Lorensia, Shahira, Prabandari, & Putri, 2023):

$$P = \int_{-\infty}^{\infty} r^2 |R(r)|^2 \, dr \tag{8}$$

This research aims to investigate the probability of the existence of He+ ions in the quantum number range 3 ≤ n ≤ 4 in momentum space. Although there are many studies reviewing the properties of hydrogenoid atoms, this work places special emphasis on analysis in momentum space, an approach that is less common but has important relevance in understanding the behavior of subatomic particles.

Method

This research is a non-experimental study that involves the transformation of the radial wave function of the hydrogen atom in position space, utilizing the equation:

$$R_{nl} = -\left(\frac{2z}{na_0}\right)^{\frac{3}{2}} \sqrt{\frac{(n-\ell-1)!}{2n[(n+1)!]^3}}\, e^{-\left(\frac{zr}{na}\right)} (\xi)^{\ell} L_{n+l}^{2\ell+1}(\xi) \tag{9}$$

In addition, the radial wave function in momentum space can also be expressed using Fourier transformation, with the equation $R(p) = \frac{1}{\sqrt{2\pi\hbar}} \int R(r)\, e^{-\frac{ipr}{\hbar}}\, dr$ By introducing new parameter, namely as follows 3 means the orbital quantum number is $l = 0, 1, 2$. So, the radial wave function for $n = 3$ is $F_{30}, F_{31} \& F_{32}$ or there are 3 radial wave functions. Meanwhile, for the main quantum number $n = 4$ are 4 here are 4 radial wave functions, namely F_{40}, F_{41}, F_{42} & F_{43}.

The following procedures are used to analytically compute the radial wave function of the helium ion in the combination of quantum numbers$(n, l) = (3, 0)$:

Determining the Gegenbauer function for$(n, l) = (3, 0)$ we get:

$$C_{n-l-1}^{l+1}\left[\frac{n^2p^2 - 4p_0^2}{n^2p^2 + 4p_0^2}\right] = C_2^1\left[\frac{9p^2 - 4p_0^2}{9p^2 + 4p_0^2}\right]$$
$$= 4\left[\frac{9p^2 - 4p_0^2}{9p^2 + 4p_0^2}\right]^2 - 1$$

The radial wave function of the helium ion

$$F_{30} = \frac{864}{\sqrt{3\pi}}\; x\; {p_o}^{\frac{5}{2}}\; x\; \frac{3\left(81p^4 - 120p^2{p_o}^2 + 16\,{p_o}^4\right)}{(9p^2 - 4{p_o}^2)^4}$$

These results are in accordance with the research results of Nurroniah et al. (2023). So the complete radial wave function of the Helium ion, He^+ can be seen in.

Table 1. Helium Ion Radial Wave Function $3 \leq n \leq 4$

n	l	$F_{n,l}(p)$
3	0	$\frac{864}{\sqrt{3\pi}}\; {p_o}^{\frac{5}{2}}\; \frac{\left(81p^4 - 120p^2{p_o}^2 + 16\,{p_o}^4\right)}{(9p^2 + 4{p_o}^2)^4}$
	1	$\frac{27648}{\sqrt{24\pi}}\; p\; {p_o}^{\frac{7}{2}}\; \frac{9p^2 - 4{p_o}^2}{\left[9p^2 + 4{p_o}^2\right]^4}$
	2	$\frac{2^{11} x\, 3^4}{\sqrt{30\pi}}\; p^2\; {p_o}^{\frac{9}{2}}\; \frac{1}{\left[9p^2 + 4{p_o}^2\right]^4}$
4	0	$\frac{2^6}{\sqrt{\pi}}\; {P_0}^{\frac{5}{2}}\; \frac{(64P^6 - 112P^4{P_0}^2 + 28P^2{P_0}^4 - 3{P_0}^6)}{\left(4P^2 + {P_0}^2\right)^5}$
	1	$\frac{2^8}{\sqrt{15\pi}} P\; {P_0}^{\frac{7}{2}}\; \frac{\left(80P^4 - 56p^2{P_0}^2 + 5p_0^4\right)}{\left(4P^2 + 4{P_0}^2\right)^5}$
	2	$\frac{2^{11}.}{\sqrt{5\pi}}\; P^2\; {P_0}^{\frac{9}{2}}\; \frac{\left(4P^4 - p^2{P_0}^2\right)}{\left(4P^2 + {P_0}^2\right)^5}$
	3	$\frac{2^{13}}{\sqrt{35\pi}}\; \frac{{P0}^{11/2}\, p^3}{\left[4p^2 + p_0^2\right]^5}$

Meanwhile, the probability of finding an electron in a helium ion is analytically done by calculating the integral of the radial distribution function of the momentum of the helium ion using the equation:

$$\mathbf{P}(\mathrm{p}) = \int_0^p p^2 \left|F_{n,l}(P)^2\right| dp \tag{10}$$

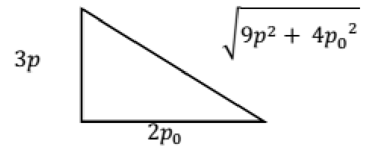
This calculation is carried out for each state in the principal quantum number 3 ≤ n ≤ 4. In the situation$(n, l) = (3,2)$, the probability of finding an electron when the electron momentum is $p = p_0 =$ 1,99285 x 10^{-24} $J.s/m$ in a Helium ion uses the following steps:

$P_{(p)} = \int_0^{9P_0} P^2 \, |F_{32}(P)^2| \, dp$

with $F_{3,2}(p) = \frac{2^{11} x \, 3^4}{\sqrt{30\pi}} p^2 \, p_o^{\frac{9}{2}} \frac{1}{\left[9p^2 + 4p_o^{\,2}\right]^4}$ obtained $P_{(p)}$

$= \frac{2^{22} 3^8}{30\pi} P_0^{\,9} \int_0^{9P_0} \frac{p^6}{(9P^2 + 4P_0^{\,2})^8} dp$

Use the Pythagorean theorem, assuming:



obtained $p = \frac{2}{3} p_0 \tan\theta$ and= $\frac{2}{3} p_0 \sec^2\theta$

By using the integration table we get P_3,2 (p_0) = 92,9938 % $P_{3,2}(p_0) = 92{,}9938$ %.

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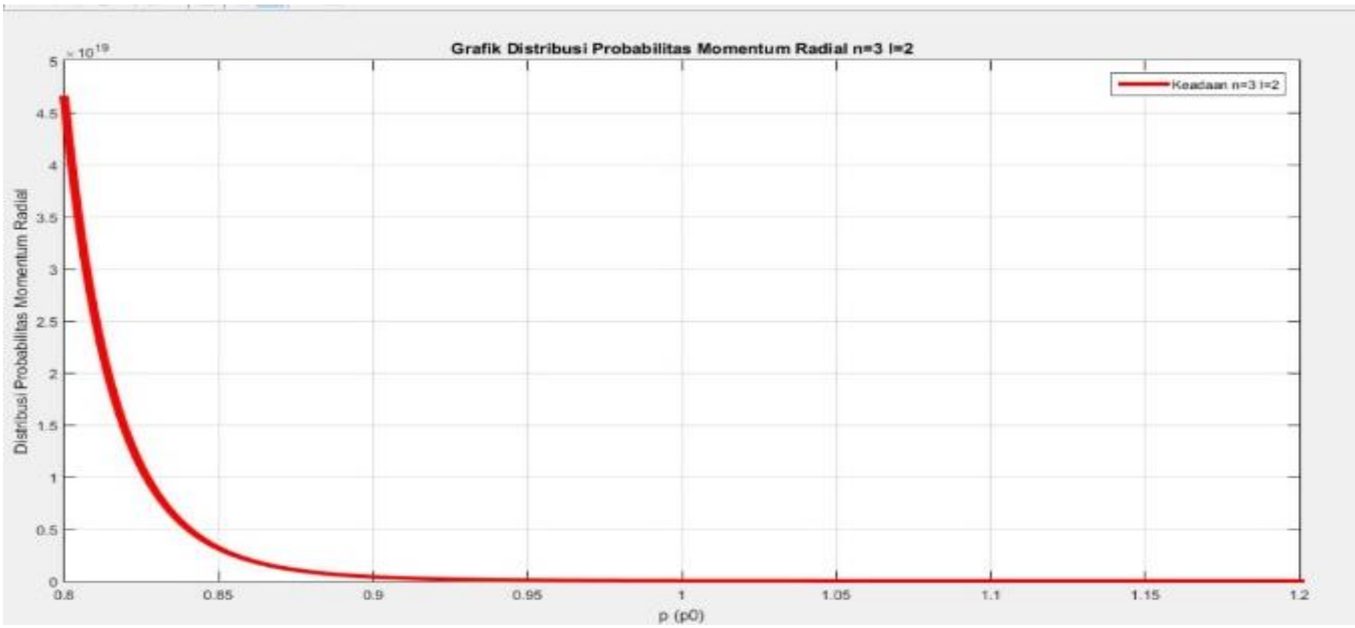

Figure 1 Radial momentum probability distribution n=3 l=2

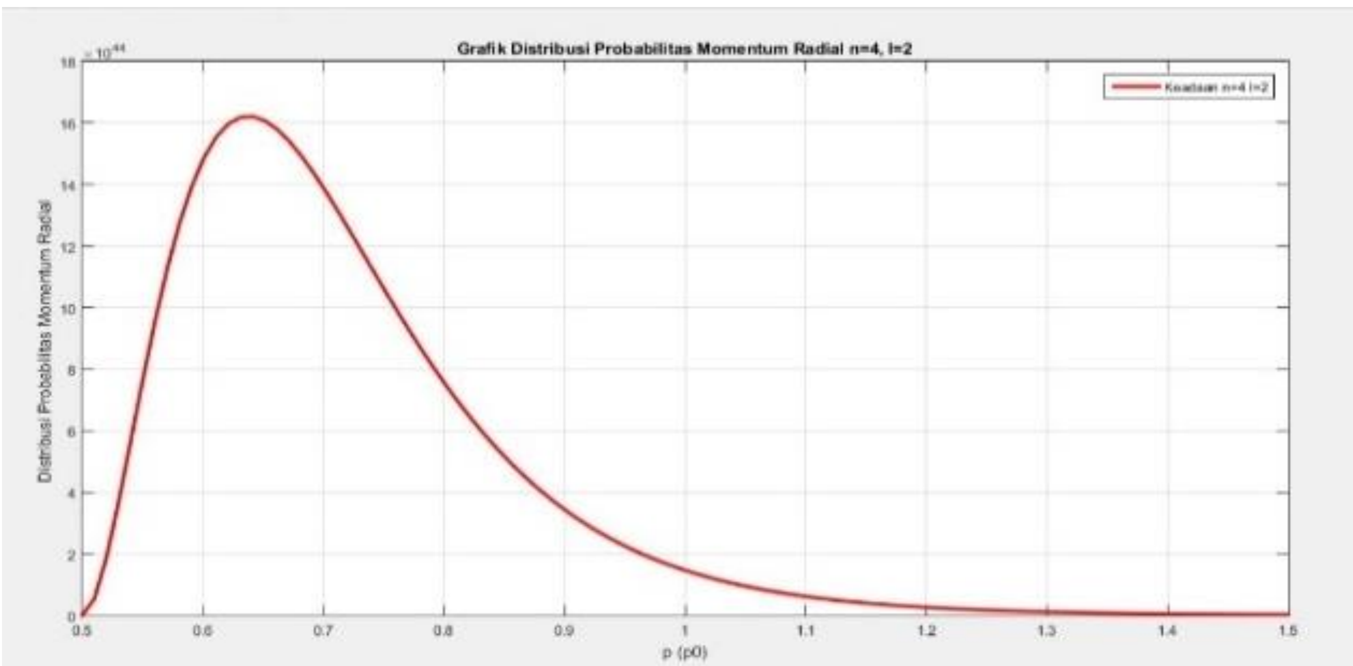

Figure 2. Radial momentum probability distribution n=4 l=2

The complete probability of finding an electron in a Helium ion at the principal quantum number $n = 3$ s presented in table 2. Apart from that, there is also an n3 l2 probability graph shown in Figure 1.

Table 2. Probability of Finding Helium Ion Electrons in the Wave Function $F_{n,l}(p)$ in the region $0 \leq p \leq 2p_0$

Interval	$F_{3,0}(p)$ (%)	$F_{3,1}(p)$ (%)	$F_{3,2}(p)$ (%)
$0 \leq p \leq 0.2p_0$	39.04	8.58	47.82
$0 \leq p \leq 0.4p_0$	61.70	59.87	16.12
$0 \leq p \leq 0.6p_0$	74.10	78.89	53.41
$0 \leq p \leq 0.8p_0$	90.49	79.94	80.86
$0 \leq p \leq 1.0p_0$	94.79	85.12	92.99
$0 \leq p \leq 1.2p_0$	95.02	90.87	97.47
$0 \leq p \leq 1.4p_0$	95.31	94.81	99.06
$0 \leq p \leq 1.6p_0$	96.02	97.11	99.64
$0 \leq p \leq 1.8p_0$	96.85	98.37	99.86
$0 \leq p \leq 2.0p_0$	97.59	99.07	99.95

Meanwhile, for the quantum number $n = 4$, the probability of finding an electron is calculated in the same way but with different integration limits. From the calculation results, it is found that the probabilities in the wave functions $F_{4,0}$ and $F_{4,2}$ are presented in table 3.

Table 3. Probability of Finding Helium Ion Electrons in the $F_{4,0}(p)$ and $F_{4,2}(0)$ Wave Functions in the region $0 \leq p \leq 0.2p_0$

Interval	$F_{4,0}(p)$ (%)	$F_{42}(p)$ (%)
$0 \leq p \leq 0.02p_0$	38.57	0.01
$0 \leq p \leq 0.04p_0$	2.95	0.58
$0 \leq p \leq 0.06p_0$	9.31	9.02
$0 \leq p \leq 0.08p_0$	20.05	60.04
$0 \leq p \leq 0.10p_0$	34.68	2.46
$0 \leq p \leq 0.12p_0$	51.81	7.38
$0 \leq p \leq 0.14p_0$	69.54	17.67
$0 \leq p \leq 0.16p_0$	85.98	35.72
$0 \leq p \leq 0.18p_0$	99.58	63.29
$0 \leq p \leq 0.20p_0$	100	100

Based on table 2 and table 3, it can be seen that the probability value for finding an electron in a Helium ion when the electron's momentum is greater, the probability is also greater. This is in accordance with the research results of Supriadi et al. (2023), namely that the probability value of the deuterium atom ${}_{1}^{2}H$ s becomes greater as the electron momentum interval and its main quantum number increase. At the same principal quantum number, the increase in the probability value for a large l orbital quantum number is faster than for a smaller l.

In the state of the main quantum number $n = 4$, the increase in probability will be faster than when $n = 3$, so in this study, the momentum interval used when $n = 4$ s smaller than when$n = 3$. This aims to see whether changes in probability values are still influenced by increasing the momentum interval used.

Conclusion

According to the aforementioned research findings, the Fourier transformation can be used to transform the radial wave function of helium ions in position space into the radial wave function of helium ions in momentum space. Then proceed with determining the integration limits using the trigonometry rule calculation technique used in calculating the probability of finding an electron in a Helium ion in momentum space, namely $p = 0$ to $p = 2\,p_0$. A helium ion's probability value is directly correlated with its electron momentum value and main quantum number. The greater the probability, the larger the electron momentum interval. This is also consistent with the findings of other hydrogenic atom research in momentum space.

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Conflicts of Interest

All authors declare that there is no conflict of interest.

References

Al Bawani, A., Supriadi, B., Syahdilla, M., Benani, N. B., & Zuhri, C. (2023). The Expectation Value of Electron Momentum of Li2+ ion on Principal Quantum Number n ≤ 3 in Momentum Space. *Jurnal Pendidikan Fisika Dan Keilmuan*, 1–7. https://doi.org/10.25273/jpfk.v9i1.16416

Alimah, S., & Sriyono, S. (2017). Kajian Sistem Pemurnian Helium Reaktor HTGR Berdaya Kecil.

Jurnal Pengembangan Energi Nuklir, *18*(2), 123. https://doi.org/10.17146/jpen.2016.18.2.3230

Berganza, C. J., & Zhang, J. H. (2013). The Role of Helium Gas in Medicine. *Med Gas Res*, *3*(1), 18. https://doi.org/10.1186/posn.9912

Bransden, B., & Joachain, C. J. (1995). *Fisika dari Atom dan Molekul*. New York: John Wiley & Sons, Inc.

Dai, D., Ismail, M. E. H., & Wang, J. (2015). Asymptotics for Laguerre polynomials with large order and parameters. *Journal of Approximation Theory*, *193*, 4–19. https://doi.org/10.1016/j.jat.2014.03.009

Hakim, L., & Nawir, M. (2016). Analisis Persamaan Schodinger Dimensi-D Bagian radial untuk Potensial Eckart Pus manning Rosen sebagai Bahan Pembelajaran Fisika Kuantum. *Jurnal Pendidikan Teknologi dan Kejuruan Balanga*, *Vol. 4 No.*(1), Hal. 30–39. Retrieved from https://e-journal.upr.ac.id/index.php/JBL/article/view/1391/1203

Kharismawati, I., & Supriadi, B. (2021). Probabilitas Partikel dalam Kotak Tiga Dimensi pada Bilangan Kuantum n≤ 5. *Jurnal Pembelajaran Fisika*, *1*(1), 58–63. Retrieved from https://jurnal.unej.ac.id/index.php/JPF/article/view/23136

Lutfin, N. A., & A, F. (2020). Metoda Hartree Fock Untuk Menentukan Energi Total Inti Helium-4. *Phydagogic Jurnal Fisika Dan Pembelajarannya*, *3*(1), 1–7. https://doi.org/10.31605/phy.v3i1.954

Marlina, E., & Rahayu, D. (2022). Desain Pembelajaran Materi Fungsi Gamma untuk Mengoptimalkan Kemampuan Berpikir Mahasiswa pada Mata Kuliah Fungsi Khusus. *Jurnal Riset Matematika Dan Sains Terapan*, *1*, 32–40. Retrieved from https://ejournal.unibba.ac.id/index.php/jrmst/article/view/787

Ningrum, T. W., Supriadi, B., Masitoh, N. N., Silvira, L., & Anggraeni, N. P. (2023). The Wave Function of The Tritium Atom in The Representation of Momentum Space Using The Scrodinger Equation on Quantum Numbers. *Jurnal Pendidikan Fisika Dan Keilmuan*, *9*(1), 23. https://doi.org/10.25273/jpfk.v9i1.16478

Nurroniah, Z., Supriadi, B., Cahyani, V. D., Salsabila, N., & Imaniyah, K. (2023). Radial Wave Function of Helium Ion in Momentum Space at N≤ 4. *Jurnal Pendidikan Fisika Dan Keilmuan (JPFK)*, *9*(1), 45–52. https://doi.org/10.25273/jpfk.v9i1.16523

Pandu, G., Pingak, R. K., Zicko Johannes, A., & Seba Ngara, Z. (2021). A Study on Radial Properties of Hydrogenic Ions using Laguerre Polynomials. *Buletin Fisika*, *23*(2), 78. https://doi.org/10.24843/BF.2022.v23.i02.p01

Plotnitsky, A. (2020). Reality, Indeterminacy, Probability, and Information in Quantum Theory. *Entropy*, *22*(7), 747. https://doi.org/10.3390/e22070747

Rillo, C., Gabal, M., Lozano, M. P., Sesé, J., Spagna, S., Diederichs, J., Sager, R., Chialvo, C., Terry, J., Rayner, G., Warburton, R., & Reineman, R. (2015). Enhancement of the Liquefaction Rate in Small-Scale Helium Liquefiers Working Near and Above the Critical Point. *Physical Review Applied*, *3*(5), 051001. https://doi.org/10.1103/PhysRevApplied.3.051001

Saputro, W. D. (2019). Rancang Bangun Alat Sterilisasi Buah Manggis Berbasis Cold Atmospheric Plasma (CAP) 2019. *Jurnal Ilmiah Teknologi Pertanian Agrotechno*, *3*(2), 313–318. https://doi.org/10.24843/JITPA.2018.v03.i02.p0

Setiyowati, R., & Riestiana, V. A. (2021). Simulasi Numerik Persamaan Gelombang Air Dangkal 1D dengan Topografi Tidak Datar Menggunakan Metode Beda Hingga. *Limits: Journal of Mathematics and Its Applications*, *18*(2), 95. https://doi.org/10.12962/limits.v18i2.6170

Supriadi, B., Anggraeni, F. K. A., Faridah, N., & Jannah, E. M. (2022). *Fisika Kuantum*. Jember: UPT Penerbitan Universitas Jember.

Supriadi, B., Lorensia, S. L., Shahira, F., Prabandari, A. M., & Putri, A. A. W. (2023). Probability of Deuterium Atom Electrons in Momentum Space at Quantum Numbers n ≤ 3. *Aceh International Journal of Science and Technology*, *12*(2), 239–245. https://doi.org/10.13170/aijst.12.2.32226

Supriadi, B., Mardhiana, H., Kristiawan, W. I., Kamalia, D., & Sari, I. K. (2023). Expected Value of Helium Ion Electron Momentum in Momentum Space with Primary Quantum Numbers n≤3. *Jurnal Penelitian Pendidikan IPA*, *9*(10), 8467–8472. https://doi.org/10.29303/jppipa.v9i10.3861

Supriadi, B., Sari, N. R., Rahmawati, E., Mardhiana, H., & Firdausyiah, W. (2023). Nilai Ekspektasi Momentum Elektron Atom Tritium dalam Ruang Momentum dengan Bilangan Kuantum Utama n ≤ 3. *Navigatatom Physics: Journal of Physics Educatatom*, *5*(1), 60–65. https://doi.org/10.30998/npjpe.v5i1.1931

Sutopo, S. (2005). *Pengantar Fisika Kuantum*. Malang: Jurusan Fisika FMIPA UM.

Suyanta, S. (2019). *Buku Ajar Kimia Unsur*. Yogyakarta: UGM Press.

Syahrial, A., Sarjan, M., Rokhmat, J., Arizona, K., Sucilestari, R., Syahidi, K., Syamsuddin, S., & Mertha, I. G. (2022). Kebermaknaan Fisika Kuantum Sebagai Solusi Membangun Karakter Peserta Didik. *Jurnal Ilmiah Profesi Pendidikan*, *7*(2b), 672–679. https://doi.org/10.29303/jipp.v7i2b.555

Utami, F., Supriadi, B., & Lesmono, A. D. (2019).

Probabilitas Posisi Elektron dalam Atom Tritium pada Bilangan Kuantum n ≤ 3. *Prosiding Seminar Nasional Pendidikan Fisika*, *4*(1), 241–245. https://doi.org/1028204/fkipe.v04ile.7497

Yanuarief, C., & Al-Faruq, A. (2019). Solusi Metode Numerik Beda Hingga Pada Visualisasi Fungsi Gelombang Persamaan Schrodinger Potensial Non Sentral Coulombic Rosen Morse. *Sunan Kalijaga Journal of Physics*, *1*(1), 28–36. Retrieved from http://ejournal.uin-suka.ac.id/saintek/physics/article/view/1631