Students' Thinking Literacy Process in Mathematical Problem-Solving

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Abstract: This study explores and investigates the process of students’ mathematical thinking literacy in solving mathematical problems. This qualitative descriptive research used a test sheet consisting of one item adapted from the PISA mathematical problem model and interview guidelines, as the instrument. One male and two female participants were selected based on problem-solving characteristics. After that, task-based interviews were carried out to investigate the process of students’ mathematical thinking literacy. Data analysis was initiated by reviewing, reducing, and concluding all collected data to describe students’ mathematical thinking literacy in solving mathematical problems. The findings suggest that mathematical thinking literacy is initiated by understanding the problem (understanding), presenting the problem in a simpler form (specializing) to represent ideas, and creating patterns or relationships (generalizing). Based on the relationship between the problem, the subject predicts a solution (conjecturing) or determines the solution to the problem, re-checks (justifying) the problem-solving process to find a logical final result, and communicates the reasons (convincing) to convince the truth of the results obtained. The results of this study help teachers understand the process of students’ mathematical thinking literacy in solving mathematical problems so that it can be used to determine the appropriate strategy in the learning process to improve students' mathematical problem-solving abilities.

Keywords: Mathematical literacy; Mathematical problems; Mathematical thinking literacy

Introduction

Education guides students to apply knowledge in daily life. Meanwhile, mathematics represents the ability to count or hone logic facilitating the solvency of daily problems, involving creativity to link ideas (Carraher et al., 2008). Therefore, mathematics education provides abilities using calculations or formulations, along with reasoning and analytical skills in solving daily problems. The recent mathematics learning process follows the standard of Mathematical Problem Solving, Communication Mathematical, Mathematical Reasoning, Mathematical Connection and representation. The students' mathematics abilities represent counting, logical and critical skills in dealing with the problem (Firdaus et al., 2019a), popular as mathematical literacy skills. Someone with great mathematical literacy is capable of applying mathematical concepts in solving everyday problems (Firdaus et al., 2019b). Therefore, mathematical lessons always aim to improve mathematical literacy, the individual's ability to formulate, use and interpret mathematics in various contexts (Stacey, 2011).

Mathematical literacy also includes mathematical reasoning and the application of mathematical concepts, procedures, facts, and tools to describe, explain and predict a phenomenon. Furthermore, mathematical literacy is related to the students' reasoning, analyzing, and communicating ideas effectively as they pose, formulate, solve and interpret deep mathematics various situations (Maryani & Widjajanti, 2020). Mathematical thinking can be interpreted as the process that allows the
Mathematical thinking is a series of dynamic processes that allow students to develop understanding and increase the complexity of the idea in solving the problem. This mathematical thinking process includes specializing, generalizing, conjecturing, and convincing (Mason et al., 2013).

Mathematical thinking is also seen as a very complex activity that involves specializing-generalizing and conjecturing-convincing. The specializing process is an activity to try, see examples or present the problem in a simpler form. The generalizing represents a process of finding patterns or relationships related to the recent problem that also facilitates the check process. Meanwhile, the conjecturing process predicts the relationship of predetermined parts, through a process of guessing, determining, using, or manipulating allegations related to the relationship of the problem, describing the identification of the solution. Lastly, convincing is a process of finding and communicating the solution. Previous studies have identified that mathematical thinking consists of four stages, namely specializing, generalizing, conjecturing, and convincing. However, no research that examines the development of those four stages leading to mathematical thinking literacy enhancement. Mathematical thinking literacy is an individual's ability to apply knowledge and skills that involve components of mathematical thinking effectively in solving problems in various contexts of everyday life.

Mathematical thinking literacy can be investigated through students' answers to the problems given. Mathematical problems induce students' desire to solve them with the correct (Al - Ghofiqi et al., 2019). As the current education goal is to accelerate students' problem-solving skills, habituation of contextual problem-solving activities helps students understand the mathematics concepts used in formal education and daily life. The contextual problems develop students' insights about the use of mathematics to solve problems in everyday life (Sa’dijah et al., 2019). Have conducted research in analyzing mathematical literacy skills that students are still unable to think mathematically literate to solve mathematical problems and problems in everyday life, so it is not enough to solve or understand PISA questions. PISA is a study developed by several developed countries join in the OECD (Almarashdi & Jarrah, 2023). Every three years, PISA monitors student learning outcomes in each participating country, including reading, mathematical, and scientific literacy. PISA's goal is to evaluate the progress of 15-year-old students in OECD countries (and other countries) in gaining appropriate proficiency in reading, mathematics, and science and their significance to their community (Stacey, 2011). While TIMSS is an international study of trends or developments in mathematics and science which is routinely carried out every four years since 1995, investigating the achievements of grade 4 until grade 8 in mathematics and science. TIMSS focuses on materials in the curriculum, such as mathematics related to numbers, metrics, geometry, data, and algebra. TIMSS is sponsored by the International Association for Education Achievement (IEA), an international association evaluating educational attainment, and its center is at the Lynch School of Education, Boston College, USA.

Indonesia's participation in PISA aims to determine the development of its national education program compared to any other country. As our next generation has to compete with another country in the era of globalization, Indonesia has been conducting PISA research from 2000 to 2018. However, a large gap between the results of the PISA study in Indonesia and the national expectations has been observed. Based on the results of Indonesia's PISA in the last 2 stages, the mathematical literacy ability of Indonesian students in 2015 was only ranked 63 out of 69 participating countries, and ranked 73 out of 79 participating countries, in 2018 (Misbah et al., 2020; Bolstad, 2023). It shows that Indonesian students have lower competency compared to other countries. Therefore, special attention from all sectors is required to improve the quality of mathematics education in Indonesia, especially related to mathematical literacy.

PISA carries out its studies every three years so that students on certain levels remain unobserved. It aims to evaluate the system education by measuring the performance of students in secondary education, especially students aged 15 years who are in the eighth grade of junior high school. Contextual problems are questions taken from real-world situations. Using contextual problems in mathematics learning enhances students' mathematics meaningful understanding, various means to apply mathematics and build mathematical models in lives (Schukajlow & Krug, 2014).

Contextual problems can be interpreted as mathematical problems formulated in such a way to represent real situations so that they become meaningful for the student. Through the use of everyday life problems, students can make mathematical models that help them develop mathematical thinking connecting the attained concept and their experiences, so that they are motivated to seek, find and build their knowledge (Harvey & Averill, 2012; Sa’dijah, 2013; Widjaja, 2013;
Mathematical thinking processes occur during the use of contextual problems. Students use their literacy skills to formulate the solvency of mathematical problems and interpret them in their real context. Previous research has been carried out to investigate the process of students’ mathematical thinking (Mason, 2008; Shafer & Foster, 1997; Uyangör, 2019) and students’ mathematical literacy (Dewantara et al., 2015; Hardianti & Zulkardi, 2019; Ojose, 2011; Oktiningrum et al., 2016; Spangenberg, 2012; Stacey, 2011).

However, only a few research focuses on the mathematical thinking literacy process of students in solving the PISA model. Therefore, this study is focused on investigating and revealing the process of mathematical thinking literacy of students in solving math problems in particular on the PISA model problem. From previous studies, there have been many researchers who have studied mathematical thinking which consists of the 4 components above: specializing, generalizing, conjecturing and convincing. But, there is still no research that examines that basically the 4 components above can still be developed and become the novelty of this research. The novelty leads to the term mathematical thinking literacy.

Method

Research Design

This study used a descriptive qualitative approach (Firdaus et al., 2020; Ikram et al., 2020; Zayyadi et al., 2020) to identify facts with the right interpretation. This study seeks to find out and reveal the process of students’ mathematical thinking literacy in solving mathematical problems, especially in the PISA model questions. The collected data were in the form of student worksheets in solving problems PISA mathematics model and interview data.

Participants

Purposive sampling was used to select three participants, consisting of one male and 2 female students, grade 9 based on the characteristics and the variety in solving the PISA-mathematical problems test. Subject 1 (S1) represented students who used way non-routine completion using a tally marks, Subject 2 (S2) represented students who answer answers the question using non-routine solutions and subject 3 (S3) represented students who used a routine solution using basic algorithms, formulas, or a simple division procedure.

Research Instruments

The instruments used in this study consisted of the main and complementary instruments. The main instrument in this study was the researcher itself as a data collector. Therefore, the presence of researchers directly in the field as a measure of success for understanding the case, so that the involvement of researchers was direct and active with informants and other data sources is required. Meanwhile, the supporting instruments was in the form of documents and other tools used to support the validity of the research results which include solving mathematical problems the PISA model developed by Novita et al. (2012), and the interview guide. The PISA-mathematical problems test consisted of 1 item that measures students’ mathematical thinking literacy skills in solving mathematical problems, especially on the PISA model questions, while the interview was carried out to obtain information reinforcement or data validity about students’ mathematical thinking literacy processes.

Research Procedure

The research procedure consisted of three stages. First, in the preparation stage, the researcher prepared the instruments, attained permission from the principal to carry out research, and coordinated with mathematics subject teachers. Second, in the data collection stage, the test sheet consisting of the PISA problem was given to students classically. The students’ answers were assessed as the basis for subject selection, research, and interviews. Third, data analysis was started by reviewing all collected data, followed by data reduction, description on the process of students’ mathematical thinking literacy in solving problems PISA mathematical models, and drawing conclusions.

Data Collection

Figure 1. The question in the test sheet about toy cars made from pomelo peels

Similarly, the data were also garnered through several stages. Firstly, the test was carried out to find the classification of the student’s answer in solving math problem-solving problems. Student answers were classified as the basis for selecting the interview subject. Interviews were conducted with subjects to confirm and explore the literacy process of mathematical thinking.
that they go through in solving mathematical problems, following the four components of mathematical thinking. Documentation in the form of important notes closely related to research was also carried out.

Data Analyze

Thematic analysis was selected to identify patterns of meaning from a set of data obtained to answer research questions because of a number of reasons. First, the theoretical framework that explains the literature on mathematical literacy thinking processes is still rare. Besides, this method offers flexibility to answer research questions. Third, this method can be used to conclude a set of data from participants and generate data patterns to identify the description of the mathematical thinking literacy process stages. Further, this method is rarely adopted in the field of mathematics education.

We followed the phase developed by Braun et al. (2006). First, we familiarized ourselves with the data, through reading and reviewing the transcript from the interview results, so that we got familiar with students' language and ideas expressed during the interview. In this process, the initial data about the process of mathematical thinking literacy were obtained. Second, in generating initial codes, we developed a preliminary code based on the results of the transcript and matched it to the six stages developed in this study. Third, looking for themes, where we created, defined, and modified code to understand interrelationships to form themes. We grouped codes that have the same meaning. Fourth, reviewing themes, where we discussed the relevance of the theme to the data repeatedly. This process was the finalization stage of the meaning of the relationship between continuity and differentiability built by students.

Fifth, defining and naming themes, where we defined and named each meaning of the relationship between continuity and differentiability that students build for graphic problems. The classification consisted of physical meaning, analytical meaning, and visual meaning. Sixth, producing the report. At this stage, we wrote the final report of the research findings. In addition, we triangulated the data to increase the objectivity of the findings, so that their trustworthiness is enhanced by the way all the authors. Also, the findings were discussed with mathematics education experts to reach a common agreement. We ensured that the data collected was accurate and complete, by administering assignments in written form and transcribing each interview immediately after recording. We also validated the coding and recoding processes of the different categories through discussions with several mathematics education experts. In the end, we found two stages of mathematical thinking literacy, namely understanding and justifying.

Result and Discussion

The students are classified according to their correct and incorrect answers. Correct answers from students are categorized into two characteristics, namely based on routine and non-routine solutions.

<table>
<thead>
<tr>
<th>Classification of Students' Answers</th>
<th>Answer Characteristics</th>
<th>The Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td>Using routine Solution</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Using non-routine Solution</td>
<td>6</td>
</tr>
<tr>
<td>Incorrect</td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1 shows a total of 19 students (61%) solve problems by routine solutions using basic algorithms, formulas, or applying simple procedures through division operations. This finding indicates that the students' mathematical literacy skills are still in the reproductive competency cluster (PISA Level 1 and Level 2). This is following the findings of (Stacey, 2011) that most Indonesian 15 years old students are still below level 2. On the other hand, 6 students (19%) solve the problems by irregularly presenting them using a certain pattern and the tally mark.

As for 6 (19%) students who give an incorrect answer, they still cannot understand or are unable to interpret the problems, and still depend on the use of certain formulas to solve problems (reproductive competence), while the mathematical problems given in this study are one level higher, namely the competency of the connection. To investigate and reveal students' mathematical thinking literacy processes in solving mathematic problems, a task-based interview with 3 selected participants was carried out. Subject 1 (S1) represents students who provide an answer using non-routine solutions using tally, Subject 2 (S2) represents students who use non-routine solutions using patterns, and Subject 3 (S3) represents students who use a routine solution using basic algorithms, formulas, or simple procedures through division operations. The following shows the literacy process of students' mathematical thinking in solving mathematical problems.

The Process of Mathematical Thinking Literacy of S1 in Solving Mathematical Problems

S1 first reads the questions and tries to understand them, then writes what is known and asked based on the provided problem. P (interviewer): What did you do the first time you get this question. S1: I read
the problem first, after that I wrote what is known. The S1’s answer sheet is presented in Figure 2.

![Figure 2](image)

**Figure 2.** The process of understanding the problems

From Figure 2, S1 simplifies the problem first by writing down the number of materials needed to make one toy car and the number of materials available for each part (sticks, peels, and car tires). Further, S1 prepares a problem-solving plan by making a tally for each material based on the available number.

![Figure 3](image)

**Figure 3.** The process of determining the number of cars carried out by S1

After making a tally for each material, S1 then classifies the tally mark based on the number of materials needed to make a toy car, the process continues until S1 gets 7 cars. P: How did you determine the number of toy cars Mr. Agus can make from the available materials? S1: I make a tally mark based on the number of available sticks available, peels, and car tires, then I mark the tally to state the number of materials used to make the cars. From the results of the interview with S1, the process of deleting the tally is illustrated in Figure 4.

![Figure 4](image)

**Figure 4.** Illustration of the process of determining the number of cars performed by S1

S1 concluded that Mr. Agus can make 7 cars, as illustrated in Figure 5.

![Figure 5](image)

**Figure 5.** Conclusions and reasons made by S1

Furthermore, S1 rechecks the results of its work to ensure no errors in the completed processes. P: After getting the results, are you sure about your answer? S1: Yes, because I have rechecked.

In solving a mathematical problem, S1’s mathematical literacy begins with understanding the problem then specializes it by writing the known information and asking questions based on the provided problems. S1 simplifies the problem by making a tally mark of every number of materials (sticks, peels, tires) available. Furthermore, S1 makes a pattern or relationship between the number of required and available materials. The process is carried out by classifying the tally which represents the materials needed to make a car consisting of 3 sticks, 2 peels, and 4 tires. The next step taken by S1 is to carry out the conjecturing process to determine the number of toy cars that can be made from available materials. S1 determines the solution to the problem by adding up the number of material groups obtained. From the conjecturing process, S1 then carries out the justification process at the rechecking stage. S1 rechecks the results of its work for ensuring no errors in any processes that have been carried out as well as conducting a convincing process by writing down the results that are also having appropriate reasons.

**The Process of Mathematical Thinking Literacy of S2 in Solving Mathematical Problems**

Mathematical problem solving by S2 starts from reading and understanding the provided problems. P: What did you do the first time you read this question? S2: I read repeatedly until I understand. S2 estimates the number of cars that can be made by Mr. Agus is 9, by only considering the number of sticks available and the number needed to make 1 car. Next, S2 tries to multiply 9 by the number of materials needed to make a car. However, the results of the
multiplication do not match the number of materials available. S2’s work sheet is presented in Figure 6.

Subject 2 performs multiplication to make sure that the estimation is correct.

Figure 6. The process of how S2 try to determine how many cars

Figure 6 shows that S2 made a calculation error in the third row after the multiplication result is obtained in the previous row which does not match the number of available materials. In the end, S2 estimation of 9 cars that can be made is inaccurate. Q: How do you determine how many toy cars Mr. Agus can make from the available materials? S2: (chuckle) I thought it was 9, it turned out to be wrong so I recalculated. S2 recalculated the answer by making a pattern or relationship between each material needed to make a toy car.

Figure 7. S2’s process of determining the number of cars

Figure 7 shows the number of materials needed to make a toy car. Column 1 states the number of sticks, column 2 indicates the number of peels, and column 3 indicates the number of tires needed to make a car. Based on the pattern or relationship made based in Figure 7, S2 concludes that Mr. Agus can make 7 toy cars from the available materials.

P: After getting the results, are you sure about your answer? S2: Yes. S2 mathematical literacy process in solving the problem starts from understanding the problem by reading the questions repeatedly. Furthermore, S2 simplifies the provided problem by presenting it in simple multiplication form. This process is a process of specializing. S2 performs the conjecturing process by estimating possible outcomes for the provided problem. Based on these estimations, S2 generalized by rechecking the answer, but the estimation is incorrect so S2 tried to recalculate by using the pattern or relationship between the numbers of the required material to make a car with the number of available materials and finally, the expected results are obtained (justifying). After obtaining the results, S2 writes the reasons to be surer of the answer gained, representing a convincing process.

The Process of Mathematical Thinking Literacy of S3 in Solving Mathematical Problems

When S1 is asked the question, S3 seems very optimistic and try to answer the question. P: What did you do the first time you read this question? S3: I’m trying to answer the question, Q: How do you determine the number of toy cars Mr. Agus can make from the available materials? S3: I share this with this (pointing to 27 and 3 in the questions), this with this (pointing to 19 and 2 in the questions), and this (pointing to 30 and 4 in the questions) the results are different. Based on the results, S3 has understood the problems given. S3’s worksheet is presented in Figure 9.

Figure 8. Conclusions and reasons made by S2

Figure 9. S3’s process of simplifying the problems

S3 performs a division operation for each type of material, by dividing the number of available materials by the number of materials needed to make a car. From the division process, S3 identifies a different quotient. To convince himself, S3 then multiplies the quotient by the number needed to make a car for each type of material. S3 concludes that the car that can be made by Mr. Agus is.
P: After getting the results, are you sure about your answer? S3: Very sure because I have proven it. In solving the mathematical problem, S3’s mathematical literacy process begins with reading and understanding the provided problems. This process is known as understanding. S3 tries to simplify (specializing) the problem by presenting it in simple division form. Through the division operation, S3 generalizes it by making a relationship between the number of available and required materials to make a car. From the division process, S3 attains different quotient results, then S3 rechecks (justifying) the correctness of the answer by multiplying the quotient by the number of materials needed to make a car based on the types of materials available. Based on results from this process, S3 carries out the conjecturing stage, to determine the answer to the provided problem. Furthermore, S3 concludes the answer and writes the reasons related to the answer.

The results of the investigation on mathematical thinking literacy of the three subjects of this study suggest a component of understanding and justification in addition to the four components suggested by previous researchers. The four components in question include specializing, generalizing, conjecturing, and convincing (Marjuwita et al., 2020; Kohen & Orenstein, 2021). The process of mathematical literacy of the subjects begins with an understanding of the mathematical problems (Taufik et al., 2019). Understanding the problem is very important in the problem solving process (Subanji et al., 2021; Sulisawati et al., 2019). This understanding generates ideas to formulate problem-solving steps. Through independent thinking, it provides space for subjects to freely express ideas in their way without having to be bound by a particular formulation or formula in solving a problem. Independent thinking occurs through a series of stages during the mathematical process (Dwivedi et al., 2023).

Each subject first reads the provided questions and tries to understand them, then simplifies it to find the relationship of each section. In this case, each subject performs a process of specializing and generalizing in estimating the problem solutions until the final result is obtained (Pascucci et al., 2023). The estimation of the solution is a conjecturing process. Further, they re-check the results so that the subjects are considered as logical answers. The re-checking process represents a justification process. Re-checking is an attempt to ensure that the completion steps and answers are correct (Sa’ dijah et al., 2021; Slamet et al., 2020). The convincing process is carried out by each subject by writing down each reason related to the answer they received as part of an effort to communicate and to assure the correctness of the answer.

Conclusion

The investigation on S1, S2, and S3 mathematical literacy process in solving mathematical problems suggests that the process of mathematical thinking literacy of each subject consists of 6 stages. It is preceded by an understanding of the mathematical problems, then the subject presents mathematical problems in a simpler form. They carry specializing processes to represent ideas and create patterns or relationships (generalizing) related to the problem. Based on the relationship between the parts in the mathematical problem, the subject estimates the solution (conjuncturing) or determines the solution to the problem. The subject re-checks (justifying) the problem-solving process to find a logical final result and communicates the reasons (convincing) to ensure the correctness of the obtained results. Thus, the literacy process of mathematical thinking of each subject occurs through a series of stages in which a mathematical process occurs.

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Author Contributions

Conceptualization; M., C. S., S., S. I.; methodology; M; validation; C. S.: formal analysis.; S.: investigation.; S. I. M; resources; C. S.: data curation: S.: writing—original; M: draft preparation; C. S.: writing—review and editing; S. I; visualization: S. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

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